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Evolution of Statistical Methods in Financial Time Series Analysis: An Empirical Analysis from ARIMA to the GARCH Family of Models

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Abstract:

Financial time series analysis is one of the key tools for asset pricing and forecasting, and its statistical methods have been continuously developing according to market demand. In the early days, the ARIMA model had great advantages in macro data analysis through its assumptions of linearity, homoscedasticity, and stationarity. Its stationary linear model was often used to predict short-term economic indicators. However, as market products gradually emerged, non-linear and highly volatile data gradually dominated the trading market, indicating that the ARIMA model was unable to characterize non-linear characteristics such as volatility clustering, asymmetric leverage effects, spike tails, and long memory, which became increasingly apparent. Therefore, the academic community has successively introduced models based on nonlinear assumptions such as ARCH, GARCH family models, EGARCH, TGARCH, and FIGARCH, which have achieved accurate prediction of financial market risks. This article conducts a literature review and comparative analysis to study the evolution, improvement, and application of ARIMA models from traditional ARIMA models to GARCH family models. It explores the advantages and limitations of ARIMA model in modeling financial time series analysis statistics, and GARCH improves the modeling framework. It summarizes the impact of its development on financial time series analysis and provides a selection of modeling methods for financial analysis.

Keywords: Financial time series analysis, ARIMA model, GARCH model, evolution of statistical methods.

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1. Introduction

The rapid development of modern financial markets has led to typical characteristics such as irregularity, asymmetry, and unpredictability in financial data, greatly increasing the difficulty of predicting future financial risks. Financial time series analysis is a key means of quantitative financial research, and whether modeling methods are scientifically developed directly affects risk management and the accuracy of asset pricing. In the early days, financial time series analysis mostly relied on modeling, among which the ARIMA model became a major method of traditional analysis due to its stable modeling ability. However, with the increasing complexity of data analysis in financial markets, the common characteristics of data volatility and heteroscedasticity in financial time series have exposed the limitations of the linear framework of ARIMA models, leading to the need for more adapted model optimization for empirical research.

From an academic perspective, financial time series analysis has evolved from linear stationary modeling to nonlinear modeling. The ARIMA model achieves sequence fitting based on the assumption of stationary time series and was widely used in early and simpler financial data analysis. To solve this problem, Engle proposed the ARCH model, which for the first time introduced the idea of conditional heteroscedasticity modeling, breaking through the theoretical constraints of traditional models [1]. Bollerslev then created the GARCH model, which improved the volatility lag fitting mechanism and became a key model for financial volatility modeling [2]. Other models that gradually emerged together formed a complete GARCH model. Using heteroscedasticity models to shape financial data can well reflect the asymmetric volatility and long-term memory characteristics of financial markets, thereby improving the research framework of financial volatility. Currently, most academia focuses on the application of individual models, lacking a systematic organization of the development and application of two models.

Based on the many shortcomings of existing research, this article has formed three progressive innovations. Firstly, existing literature mainly focuses on fragmented analysis of single ARIMA or GARCH sub-models, lacking a systematic sorting of the complete iterative process of the two types of models. This article takes the transformation from linear modeling to non-linear heteroskedasticity modeling as the main line, sorts out the complete evolution path from ARIMA to various extended GARCH models, and constructs a complete theoretical analysis framework for financial time series models from linear to nonlinear; Secondly, existing research has not established a model comparison standard for different scenarios. This article distinguishes between two research scenarios: predicting low-volatility macroeconomic indicators and high-frequency nonlinear stock markets. It compares the predictive performance of ARIMA and GARCH family models, clarifies the differential applicability boundaries of the two types of models, and provides theoretical reference for model selection in empirical modeling; Thirdly, previous research has separated the logic of model improvement from the inherent statistical characteristics of financial data. This article focuses on the analysis of four nonlinear features: volatility clustering, asymmetric leverage effect, peak fat tail, and volatility long-memory. It explains the core advantages of GARCH family models in adapting to heteroscedastic financial data, expands financial time series research from mean trend prediction to volatility risk quantification, and provides theoretical ideas for the development of mixed time series models in the future.

2. The Evolution Background of Statistical Methods for Financial Time Series Analysis

Financial time series modeling is a core foundational tool for asset pricing, risk management, and quantitative forecasting. Early financial markets were dominated by stable and easily analyzable data, with trading varieties concentrated on low-frequency macro indicators and traditional fixed-income products. Price fluctuations were gentle, and extreme market shocks were minimal. ARIMA models became representative tools for linear stationary time series modeling, laying a solid foundation for financial time series analysis methodology.

With the continuous development of the financial market, market disturbances caused by macro shocks, policy regulation, and unexpected events continue to intensify. Derivatives, high-frequency trading of stocks, and cross-border capital flows greatly complicate the logic of market operation. Financial time series data generally exhibit four typical non-linear characteristics: volatility clustering, heteroscedasticity, spike and fat tails, and asymmetric volatility. For example, market panic selling triggered by negative policies can cause sustained volatility, reflecting the characteristics of volatility clustering; During the crisis phase, a significant price gap can lead to a peak in yield and a fat-tailed distribution; The market's response to negative news is much stronger than that of positive news, resulting in significant asymmetric volatility. These characteristics fundamentally contradict the three core assumptions of ARIMA model: linearity, homoscedasticity, and stationarity. In this context, the academic community gradually broke through the constraints of linear frameworks: Engle was the first to propose ARCH models to solve the heteroscedasticity problem, and subsequent scholars further expanded and formed a complete GARCH family system, becoming the mainstream research direction of modern financial econometrics.

This chapter uses literature review, comparative analysis, and empirical induction as the basic research methods to sort out the classic literature on linear and time-varying variance time series models, compare the applicability boundaries of the two frameworks under different market data, summarize the empirical conclusions of existing financial modeling, and systematically sort out the relevant research on financial time series analysis methods, completing the horizontal comparison between ARIMA models and GARCH family models.

3. Model Introduction

3.1 Principles and Digital Structure of ARIMA Model

The ARIMA model was proposed by Box and Jenkins and evolved through the progressive evolution of AR, MA, and ARMA models [3]. It is a classic time series prediction model that is based on the assumption of constant variance and characterizes the linear variation of first-order moments in time series.

3.1.1 AR autoregressive model

The AR autoregressive model, as a fundamental model in univariate stationary time series, is used to describe the linear relationship between the values of the current series and its own p-order lag term. Stable AR sequences already have the characteristic of mean reversion, which means that external shocks only temporarily affect them. Over time, the sequence will inevitably return to its average value. However, due to this phenomenon, false regression and parameter distortion may occur over time. Therefore, only stable sequences will not experience such situations. The prediction result of AR model has statistical significance.

3.1.2 MA moving average model

The MA moving average model only uses a linear composite sequence of white noise disturbances in the current and past q periods, which has a certain degree of stationarity and is conducive to identifying unique parameters and avoiding multiple solutions. However, it only stores the current noise level of q and cannot effectively distinguish possible future fluctuations, so it can only temporarily have effective predictive ability.

3.1.3 ARMA differential autoregressive moving average model

The ARMA model models AR historical sequences and MA historical noise, with much higher fitting ability than standalone AR or MA models, and is often used for stationary univariate time series prediction. However, the AR feature equation requires all root modes to be greater than 1, and the MA feature equation also requires all root modes to be greater than 1. This core requirement is to accurately predict stationary variables, and it is completely useless if non-stationary variables are encountered.

3.1.4 ARIMA (Autoregressive Moving Average) model

The ARIMA model handles non-stationary univariate series with unit roots. It works by differencing the stationary series by order d and then fitting the result to the stationary series. This model is suitable for non-stationary linear time series forecasting without seasonal variations. However, it only decomposes the stochastic trend and short-term disturbances, without decomposing the long-term trend. This makes it unable to handle seasonal fluctuations and carries the risk of fitting distortion.

3.1.5 ARIMA Model standardization steps

Data Preprocessing and Stationarity Test: Clean missing values and remove extreme outliers. Use the ADF unit root test to determine the stationarity of the sequence. If the p-value is greater than the significance level, the sequence is considered non-stationary.

Determining the Difference Order d: Difference the sequence successively, performing the ADF test again after each difference until the sequence is stationary. The number of differences is the optimal d. For financial price sequences, the optimal differential order d is usually set to 1.

Lag Order Identification: Plot the ACF and PACF diagrams of the stationary sequence. Combine the AIC and BIC information criteria to select the optimal p and q combination.

Model Parameter Estimation: Maximum likelihood estimation is used to solve for constants, autoregressive coefficients, and moving average coefficients.

Residual White Noise Test: Use the Ljung-Box test on the residual sequence [4]. If the residuals are white noise, it indicates that the linear information in the sequence has been sufficiently captured. If not, the lag order needs to be

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readjusted.
Short-Term Trend Forecasting: Based on the fitted model, perform mean forecasting and output the predicted short-term trend level.

3.2 GARCH Family Models: Volatility Modeling

Because the ARIMA model cannot represent heteroscedasticity, Engle proposed the ARCH model, achieving dynamic modeling of conditional variance for the first time. Building on this, Bollerslev proposed the GARCH family of models, solving the problem of excessive parameters in the ARCH model. Later, scholars raised questions about phenomena such as long memory, leading to further disagreements. Branch models such as EGARCH, TGARCH, and FIGARCH constitute the complete GARCH family of models, frequently used to predict financial market risk.

3.2.1 The theoretical leap from ARCH to GARCH

The ARCH model treats the current conditional variance as linearly determined by the squared terms of historical residuals, thus breaking the homoscedasticity assumption. However, its application in financial markets still easily encounters problems such as insignificant parameters and multicollinearity, resulting in very low empirical fitting efficiency. The GARCH model made a breakthrough improvement based on this foundation. Bollerslev added the lagged conditional variance term to the ARCH framework. This allows the model to use a small number of higher-order parameters to represent the persistence of long-term volatility, greatly reducing the difficulty of parameter estimation and accurately representing the concentrated characteristics of volatility in financial markets [5].

3.2.2 Expanding the GARCH family of models

The EGARCH model creates the variance equation in the form of exponential GARCH, thus solving the problem that variance cannot be negative. The model uses an exponential form to distinguish the differentiated effects of positive and negative shocks, successfully avoiding the parameter sign restriction problem of GARCH. Because it is suitable for quantifying the volatility surge caused by extreme negative news, it is suitable for high-volatility markets such as cryptocurrencies and forex.
Zakoian uses a threshold indicator function to identify market leverage effects, thereby distinguishing the impact of positive and negative shocks on volatility [6]. TGARCH is suitable for equity markets such as stocks and stock indices, and will conduct special research on asymmetric volatility caused by sharp stock market declines and negative policies, and will also show the situation of rapid increase in volatility under a downtrend.
Bollerslev & Mikkelsen use fractional difference operators to break through the limitations of integer difference, in order to represent the long memory characteristics of volatility and obtain the long-term impact of distant historical shocks on current volatility [7]. The FIGARCH model is suitable for medium- and long-term financial time series modeling, such as commodity and long-term stock index volatility research. There is a situation where volatility decays very slowly and has long memory characteristics.

4. Empirical Comparison of Models and Typical Viewpoints in the Same Financial Scenario

4.1 Macroeconomic Indicator Prediction.

The ARIMA model can achieve the highest accuracy in prediction in the short term (1-3 months) and is the basic model of academia and central banks. Ahmar pointed out that the linear framework of ARIMA can accurately grasp the data in the macroeconomy that has obvious trends and small fluctuations, and has strong explanatory power, which is conducive to policy analysis [8]. However, the GARCH family model is far weaker than the ARIMA model due to its complicated introduction method. The empirical study of Nwosu, C. O., & Okonkwo, S. I. proposed that the heteroscedasticity of macro data is not obvious, and the accuracy of the GARCH family model is generally low [9]. Therefore, in the macro scenario, the accuracy of the ARIMA model is much higher than that of the GARCH family model [10].

4.2 Stock Market Analysis.

On the contrary, in this case, the ARIMA model has only a rough effect on short-term price prediction, and the volatility prediction is completely ineffective. Misha, D. repeated the study of Andersen & Bollerslev on the S&P 500 index, showing that the volatility prediction R^2 of the ARIMA model is less than 5%, and it cannot distinguish the effects of good news and bad news [11]. When the market falls sharply, its prediction deviation can reach more than 300% [12]. However, the GARCH family of models is very accurate in predicting volatility. Nwosu, C. O. found that the accuracy of TGARCH in describing asymmetric volatility in A-shares and US stocks is more than 20% higher than that of the basic GARCH [13,14]. Therefore, the GARCH family of models is suitable for high-frequency, nonlinear situations [15].

5. Core Differences and Comments of the Model

The essential differences between ARIMA models and GARCH family models are reflected in two aspects: first, they rely on different theoretical assumptions, and second, they are suitable for different types of data. The ARIMA model works based on the assumptions of linearity, homoscedasticity, and stationarity, with a focus on depicting the trend of the sequence mean. However, the GARCH family models are analyzed based on the assumptions of nonlinearity and conditional heteroscedasticity, emphasizing the dynamic characteristics of volatility, thereby promoting financial time series research from “trend estimation” to a higher level of “volatility and risk quantification”.

Most of the existing relevant studies focus on the empirical test or partial improvement of a single model, and few systematically sort out the research on the evolution process, evolution reasons, application scenarios and application effects of ARIMA to GARCH family models, which makes it impossible for the article to clearly show the model iteration method, process and reasons. At the same time, there are significant shortcomings in the academic community’s comparative research on the two models in different environments, scenarios, and data. Furthermore, there are still different controversies in the integration and evolution with new technologies, and effective academic unity has not been achieved [16,17]. Based on the above research gaps, this article decides to systematically sort out the complete evolution of financial time series modeling from linear stationarity to nonlinear heteroscedasticity, provide theoretical references for the selection of methods for financial time series modeling and subsequent research, and form a clear logical framework.

6. Conclusion

The article outlines the entire process of the transition from linear ARIMA models to GARCH family models in financial time series, compares the practicality and effectiveness of these two models in various situations, and discusses their respective shortcomings. ARIMA’s model architecture is simple and clear, and its economic significance is also relatively intuitive. It belongs to the traditional linear time series method. When encountering market data with small fluctuations and moving towards stability, the prediction error made in the short term will be smaller than other methods. Therefore, it is widely used in low-volatility indicators such as GDP and inflation. However, due to its key assumptions of linearity, homoscedasticity, and normal distribution, it cannot adapt to the nonlinear characteristics of financial data in today’s market, such as concentrated volatility, asymmetric leverage, and thick tails. It is easy to have inaccurate fitting.

Firstly, the article only provides a systematic analysis of ARIMA and GARCH family models, and does not touch upon the development of other financial time series models, such as the SV and the Copula models, and this paper fails to discuss how these alternative models interact with ARIMA and GARCH frameworks, so the analysis in the article has certain limitations. Moreover, the article is relatively short and did not conduct specific empirical tests to ensure the good or bad performance of the model in practical applications. In the future, various empirical studies can be conducted to improve the overall content of this article. For further research, the combination of the GARCH family and deep learning can be emphasized to explore new hybrid models suitable for high-frequency trading and extreme risk scenarios, and to increase the empirical comparative research on different models in emerging markets, to determine the optimal applicability range of each model.

Looking to the future, this article also leaves many valuable directions for further research. Firstly, subsequent research can incorporate other mainstream financial time series models such as SV and Copula into comparative analysis frameworks, and construct a complete model evaluation system that breaks through the limitations of ARIMA and GARCH families and covers more tools; Secondly, targeted empirical tests can be conducted based on real market data of stocks, foreign exchange and bulk commodity markets, quantifying the differences between the actual fitting and prediction effects of various models to compensate for the lack of independent empirical testing in this article; In addition, researchers can integrate GARCH family models with deep learning algorithms to construct a hybrid volatility prediction model that is suitable for high-frequency trading and extreme risk scenarios; Simultaneously conducting cross-market comparative empirical analysis on the financial markets of emerging economies, further refining the optimal application scenarios and boundary conditions for various time series models.

References

[1] Engle R F. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 1982, 50(4): 987-1007.

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[2] Bollerslev T. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 1986, 31(3): 307-327.
[3] Hamilton J D. Time Series Analysis. Princeton University Press, 1994.
[4] Ljung, G. M., & Box, G. E. P. . On a measure of lack of fit in time series models. Biometrika, 1978, 65(2): 297–303.
[5] Bollerslev T, Engle R F, & Nelson D B. ARCH models. Handbook of Econometrics, 1994, 4: 2959-3038.
[6] Zakoian J M. Threshold heteroskedastic models. Journal of Economic Dynamics and Control, 1994, 18(5): 931-955.
[7] Bollerslev T, & Mikkelsen H O. Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics, 1996, 74(1): 3-30.
[8] Ahmar A S. Novel time series methods in economic forecasting: SutteARIMA evidence from Indonesian Consumer Price Index. Journal of Applied Statistics, 2025, 52(14): 2789-2806.
[9] Nwosu C O, & Okonkwo S I. Comparative Analysis of stochastic process in forecasting key macroeconomic variables in Nigeria. Lagos Journal of Management and Administration, 2025, 3(2): 31-47.
[10] Ho S T, & Lee J H. ARIMA vs. Machine Learning in Portfolio Return Forecasting: A Comparative Study Integrating GARCH-Based Volatility Estimation. In Proceedings of the 15th International Conference on Economics, Finance and Statistics. Scitepress, 2025: 412-419.
[11] Misha D. An Analysis of the Predictive Ability of ARIMA-GARCH Models on the S&P 500 Index During Crisis Periods. In Proceedings of the 17th International Conference on Financial Econometrics. Atlantis Press, 2025: 1046-1053.
[12] Andersen T G, & Bollerslev T. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. International Economic Review, 1998, 39(4): 885-905.
[13] Nwosu C O. A HYBRID APPROACH TO FORECASTING STOCK INDICES USING THE ARMA–GARCH AND ARMA–EGARCH MODELS: Evidence from the Nigerian Stock Exchange. Lagos Journal of Management and Administration, 2025, 3(1): 1-22.
[14] Kaur P. A Comparative Modelling of Essential Characteristics of Volatility. Journal of Applied Financial Econometrics, 2025, 7(1): 33-47.
[15] Li M. A study of financial time series volatility forecasting method based on GARCH modeling. In ACM 2024 International Conference on Big Data Finance. ACM Press, 2024: 112-118.
[16] Mokoena T, & van der Merwe L. A Fusion of Statistical and Machine Learning Methods: GARCH-XGBoost for Improved Volatility Modelling of the JSE Top40 Index. Mathematics, 2025, 13(3): 155.
[17] Okafor I. Developing the Hybrid ARIMA-FIGARCH Model for Time Series Analysis. International Journal of Financial Econometrics, 2025, 8(2): 56-71.