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High-Dimensional Granger Causality with GARCH-Filtered Returns: Risk Spillovers Modeling

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Abstract:
Stock markets are complex systems in which volatility and shocks can propagate rapidly across assets. Therefore, the perception of correlation between stocks is essential for identifying volatility transmission paths and systemic risks. However, the conventional Granger Causality Test is sensitive to the heteroskedastic and heavy-tailed nature of asset returns. This paper integrates the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model with the Post-Double-Selection (PDS) to study how shocks transmit across assets. Each return series is first fitted with the GARCH model, and the conditional variances are utilized to isolate Granger causalities across assets. The framework extends traditional causality analysis to actual financial systems, providing a new perspective on risk transmission and volatility spillovers. The empirical results show that the proposed GARCH-PDS Granger Causality approach substantially improves the detection of contagion pathways and enhances the interpretability of systemic interdependencies, with limited sensitivity to the dimensionality of data. These findings contribute to the econometric modeling of causal linkages across assets and provide practical implications for macroprudential risk monitoring.

Keywords: GARCH model; granger causality test; high-dimensional inference; post-double-selection.

1. Introduction

Persistent volatility, strong interconnections, and rapid transmission of shocks across assets and sectors characterize financial markets. Episodes such as the global financial crisis or the Coronavirus Disease 2019 (COVID-19) pandemic have demonstrated how localized disturbances can escalate into systemic events. Traditional econometric tools are often unable to cope with high dimensionality and heteroskedasticity simultaneously. Under such circumstances, uncovering the direction of information flows is crucial to risk monitoring, portfolio allocation, and policy design.

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A predictive definition of causality based on the idea that past values of one variable can help forecast another was proposed [1]. Building on this idea, the vector autoregressive (VAR) framework was developed, which became the standard tool for empirical causality analysis [2]. This widely used framework provided a basis for testing directional relationships in economic and financial time series. However, standard Granger Causality Tests assume low-dimensional settings and quickly lose reliability when the number of variables increases. Several recent studies have extended causality testing to high-dimensional environments. A Post-Double-Selection (PDS) procedure was introduced, which was based on least absolute shrinkage and selection operator (LASSO) variable selection for Granger Causality Test in high-dimensional VAR models [3]. Apart from that, a network-oriented framework constructs debiased-LASSO test statistics that identify statistical causality links within complex financial networks [4]. By combining lag augmentation with variable selection, their method allows robust inference even when the integration orders of the time series are unknown or when cointegration relationships exist. In addition, a high-dimensional Granger Causality (HDGC) Test was also introduced, which integrates sparse-group LASSO regularization with debiased inference [5]. Furthermore, methods for variable-lag Granger Causality have been proposed, allowing causal effects to occur at flexible time lags [6]. Moreover, quantile-based Granger Causality has been applied to capture both linear and nonlinear causality [7].

The econometric modeling of volatility began with autoregressive conditional heteroskedasticity (ARCH) model, which captured volatility clustering in inflation [8]. This model was soon extended into the GARCH model [9]. Network-based GARCH models have been proposed to account for the interconnected structure of financial assets, which introduces structured sparsity that reduces parameter dimensionality [10]. Furthermore, the Realized-GARCH family has integrated high-frequency information into conditional volatility modeling. The Realized-GARCH framework has recently been applied to both traditional financial assets and emerging markets such as cryptocurrencies, demonstrating its flexibility and robustness in high-frequency environments [11]. At the same time, several studies have re-evaluated the robustness and predictive performance of classical versus extended GARCH specifications [12].

These developments in causality testing and volatility modeling have paved the way for integrated approaches that analyze dependence structures and volatility dynamics. In this spirit, the present study integrates GARCH-based volatility filtering with high-dimensional Granger Causality analysis to examine how shocks are transmitted across financial assets. Rather than aiming to pinpoint individual causal links, the analysis seeks to reveal the broader pattern of interdependence that characterizes systemic risk.

2. Method

2.1 GARCH Model

While the squared daily return r_t^2 can serve as a simple estimator for actual volatility, it is widely recognized as being unbiased but highly inefficient [13]. To obtain a smoother and more precise measure, this study employs the GARCH model. This model is the standard approach in financial econometrics as it effectively captures volatility clustering and leptokurtosis of asset returns.

2.1.1 Model specification

The GARCH model consists of a mean equation and a variance equation. Let r_t be the logarithmic return series of a stock, defined as:

$$r_t = \ln(P_t) 0 \ln(P_{t-1}) \tag{1}$$

where P_t denotes the closing price of the stock on day t.

The mean equation is specified to filter out any linear autocorrelation in the return series:

$$r_t = \mu_t + \varepsilon_t \tag{2}$$

Here, μ_t is the conditional mean of the returns, and $?_t$ is the error term, which can be decomposed as:

$$\epsilon_t = Z_t \sigma_t \tag{3}$$

In this equation above, $\{Z_t\}$ is a strict white noise process, which is a set of independent and identically distributed (i.i.d.) random variables with zero mean and unit variance, and has an unknown marginal distribution function $F_Z(z)$. Meanwhile, μ_t and σ_t are respectively the conditional mean and conditional standard deviation, and both are measurable about the information set G_{t-1}, which represents the set of all observable returns up to day $t-1$.

The variance equation is the core of the GARCH model. This study adopts the most classic and widely used GARCH(1,1) specification, with the variance equation defined as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{4}$$

where σ_t^2 represents the conditional variance at time t, ϵ_{t-1}^2 represents the squared shock from the previous period, and parameters, α_0, α_1, β, should be greater than zero.

2.1.2 Model application

In this study, a GARCH(1,1) model is individually fitted to the daily log-return series of each stock in the sample. After ensuring that each model has successfully converged, it extract its conditional variance series, $\{\sigma_t^2\}$, and logarithmic transformation is applied to the series. This time series provides a precise and robust measure of daily volatility for each stock. Subsequently, these generated volatility series are used as the core input variables for the high-dimensional VAR model to investigate the causality of the volatility in the stock market.

2.2 Granger Causality Test with Post-Double-Selection

Granger proposed a method to describe causality from the perspective of prediction [1]. This concept is that the accuracy of predicting the future value of a time series can be improved based on the observed values of another series. Given the information set Ω and time series y_t, another time series x_t is observed when Ω and y_t are known. At this time, information on Ω and y_t can be applied to estimate x_t at time t. If the variance of the estimation error of x_t, with y_t involved, is different from that when using the historical information of Ω alone, the obtained sequence y_t can be considered to improve the predictability of the time series x_t. The process above can be expressed as Equation 5:

$$\mathrm{var}\left(F\left(X_t|\Omega_{<t}\bigcup Y_{<t}\right)0X_t\right) < \mathrm{var}\left(F\left(X_t|\Omega_{<t}\right)0X_t\right) \quad (5)$$

where X_t denotes the real value of x_t at time t, $\Omega_{<t}$ and $Y_{<t}$ represent the obtained information of set Ω and the value of y_t until time $t\text{-}1$, respectively. $F\left(X_t|\Omega_{<t}\right)$ can be regarded as the optimal estimation of x_t at time t, with $\Omega_{<t}$ obtained exclusively.

However, improved prediction does not represent true causality, but rather a statistical measure of information flow. In fact, Y is said to be the Granger-cause of X if lagged values of Y provide incremental predictive content for X, conditional on the past of X. When volatility is quantified by the GARCH conditional variance and applied as a test variable, the resulting causal relationships reflect the transmission paths of risk. When constructing a volatility spillover network composed of a large number of stocks, the traditional VAR model faces a severe „Curse of Dimensionality" problem. A standard VAR model with a lag order of p requires the estimation of $N(Np+1)$ parameters. When the number of stocks N is large (in this study, $N=40$), the number of model parameters expands rapidly, leading to unstable estimation results, overfitting, and unreliability. Therefore, the standard VAR model is not feasible in this high-dimensional scenario.

A simple alternative is to perform $N(N\text{-}1)$ pairwise Granger Causality Tests. A bivariate Granger Causality Test between x_t and y_t with k lags based on VAR model can be defined as:

$$a_x^0 x_t = \sum_{k=1}^{d} a_{xx}^k x_{t\text{-}k} + \sum_{k=1}^{d} a_{xy}^k y_{t\text{-}k} + e_{t,x} \quad (6)$$

$$a_y^0 y_t = \sum_{k=1}^{d} a_{yy}^k y_{t\text{-}k} + \sum_{k=1}^{d} a_{yx}^k x_{t\text{-}k} + e_{t,y} \quad (7)$$

where a (indexed by subscripts and superscripts) and e (indexed by subscripts) represent the corresponding model coefficients and error terms, respectively. Under such circumstances, y_t can be identified as the Granger-cause of x_t, if and only if there exists a k $\left(1\leqslant k\leqslant d\right)$ such that $a_{ij}^k \neq 0$. A hypothesis test can be used to identify Granger causality: H_0: $a_{xy}^1=a_{xy}^2=\cdots=a_{xy}^d=0$ vs. H_1: $\exists a_{ij}^k \neq 0$

However, this method overlooks the joint influence of other stocks within the system. It is highly susceptible to omitted variable bias, resulting in a large number of spurious relationships and an inability to depict the true and direct spillover paths accurately.

Consider the high-dimensional Granger Causality Test in VAR model based on the definition of Equation 8:

$$x_t = \sum_{k=1}^{d} A^k x_{t\text{-}k} + e_t \quad (8)$$

where $x_t=\left(x_{1t},x_{2t},¡,x_{pt}\right)^T$ denotes the variable at time t, e_t is a white noise term of dimension p, $A^0,A^1,¡,A^d$ represent is the coefficient matrix of order $p¡Áp$ and d is the order of the time series. The necessary and sufficient condition for sequence x_i to become the Granger cause of sequence x_j is that there exists a k $\left(1\leqslant k\leqslant d\right)$ such that $A_{ij}^k \neq 0$.

In the HDGC test, it is possible to use the regularization method to select the variables and then use the obtained variables for ordinary least squares estimation. However, the final model obtained by simple one-step regularization is a function of the input data. The selection process will introduce additional uncertainty into the final statistics. This uncertainty arises from the fact that the direct use of regularization methods may excessively remove weakly correlated variables, leading to omitted variable deviation.

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To overcome the above challenges, this study employs the Post-Double-Selection Granger Causality Test. This method combines LASSO penalized regression with PDS theory [3]. It can effectively perform variable selection in high-dimensional systems and make reliable statistical inferences on the filtered variables.

2.2.1 LASSO regression and post-double-selection

The LASSO regression is a penalized linear regression. By adding a penalty term to the objective function, it can compress some unimportant variable coefficients to equal zero strictly, so as to realize variable selection and model sparseness [14].

When performing tests on HDGC model, let the penalty function for the coefficient matrix be $\sigma(?)$, the objective function for parameter selection can be defined as:

$$\min_{A^1,A^2,\dots,A^d\in R^{p\times p}} \sum_{t=d+1}^{T} x_t - \sum_{k=1}^{d} A^k x_{t-k2}^{\;2} + \sigma\left(A^1,A^2,\dots,A^d\right) \quad (9)$$

where $¡¬?¡¬_2$ represents the 2-norm of the matrix, and T is the length of the time series.

Among them, the elements in the coefficient matrix are compressed to zero by adjusting parameters for selection, and the penalty function can be written as:

$$\sigma\left(A^1,A^2,¡,A^d\right)=\lambda\sum_{k=1}^{d}\sum_{j=1}^{p}\left|A_{ij}^k\right| \quad (10)$$

where sequence x_i is the Granger cause of sequence x_j, if there exists a certain k $\left(1\leqslant k\leqslant d\right)$ such that $A_{ij}^k\neq 0$.

However, the standard LASSO regression introduces errors into the estimates of the other non-zero coefficients, leading to the failure of the traditional significance test.

A PDS procedure was proposed when constructing a partial linear model to facilitate consistent inference of effects [15]. The basic idea is to carry out two-step LASSO regression first, and then OLS regression. Specifically, LASSO regression is performed on the output and processing variables with respect to all control variables. After the two-step selection step, the output variable is regressed with respect to the processing variable and the control variable whose coefficient is not zero in at least one LASSO regression. Through the two-step selection, the main regression can effectively reduce the deviation caused by missing variables, so that the error term of the final model can be considered orthogonal to the main processing variables at a sufficient confidence level. At the same time, the information provided by weak correlation is retained to a certain extent. The logic of selection has been extended to regression in high-dimensional systems based on the above estimation and inference, allowing the data to exhibit weak correlation in both time series and cross-sectional dimensions [16].

2.2.2 Method

Let X_{GC} denotes the variable matrix that may cause Granger causality, and X_{-GC} denotes the matrix excluding variables that may cause Granger causality, $X_{GC}^{?}$ and $X_{-GC}^{?}$ can be defined via the Kronecker product

$$X_{GC}^{?}=E_{N_I}?X_{GC} \quad (11)$$

$$X_{-GC}^{?}=E_{N_I}?X_{-GC} \quad (12)$$

where E is an identity matrix, N_I represents the number of elements in set I, and I is the set of labels of dependent variables to be tested.

The key to introducing the concept of PDS in conducting Granger Causality Test is to incorporate a two-step variable selection regression consisting of Equation 13 and Equation 14. Let γ_j denotes coefficients and e_j stands for the error term,

$$y_I=X_{-GC}^{?}\gamma_0+e_0 \quad (13)$$

$$x_{GC,j}=X_{-GC}\gamma_j+e_j,j=1,¡,N_X \quad (14)$$

where the subscript GC denotes the independent variables that may exhibit Granger causality in the desired study, y represents the dependent variables that may be affected, and the subscript $-GC$ indicates the remaining variables. N_X is the number of columns for potential Granger causal variables. Equation 13 represents the identification of the relationship between variables that do not participate in the Granger Causality tests and the target dependent variable, and Equation 14 represents the identification of the relationship between variables that do not participate in the Granger Causality tests and the target independent variable. Since Equations 13 and 14 are still multidimensional or high-dimensional systems, Lasso estimation is used here for variable selection and identification.

Let labels of variables with non-zero regression coefficients generated by Equation 13 and 14 as set $\hat{S}_0$ and $\hat{S}_j$ respectively. Let $\hat{S}_X=\bigcup_{j=1}^{pN_j}\hat{S}_j$, $\hat{S}=\hat{S}_X\bigcup\hat{S}_0$, then eliminate variables with zero regression coefficients in $X_{-GC}^{?}$ to get $X_{\hat{S}}^{?}$. On this basis, an ordinary least square (OLS) can be implemented:

$$y_I=X_{\hat{S}}^{?}\beta_I+\xi \quad (15)$$

and denote the matrix which consists of residual vectors as $\hat{\Xi}$. Let $\hat{\Sigma}=\hat{\Xi}¡®\hat{\Xi}/T_d$ where T_d denotes the overall number of periods of data to be tested. $\hat{\Sigma}^{?}$ can also be defined as the Kronecker product:

$$\hat{\Sigma}^{?}=\hat{\Sigma}?E_{(T_d-p)} \tag{16}$$

and $\left(\hat{\Sigma}^{?}\right)^{-1/2}$ lays the foundation of the following OLS regressions.

Let $y_I^*=\left(\hat{\Sigma}^{?}\right)^{-1/2}y_I$, $X_{GC}^{?*}=\left(\hat{\Sigma}^{?}\right)^{-1/2}X_{GC}^{?}$ and $X_{\hat{S}}^{?*}=\left(\hat{\Sigma}^{?}\right)^{-1/2}X_{\hat{S}}^{?}$. Two more OLS regressions can be performed:

$$y_I^*=X_{\hat{S}}^{?*}\beta_2+\xi^* \tag{17}$$

$$y_I^*=X_{GC}^{?*}\beta_3+X_{\hat{S}}^{?*}\beta_4+v \tag{18}$$

and denote residuals derived from each regression as $\hat{\xi}^*$ and $\hat{v}^*$ respectively. Under this circumstance, LM can be calculated as the test statistic and can be derived from the following formula:

$$LM=\hat{\xi}^{*'}\hat{\xi}^{*}0\hat{v}^{*'}\hat{v}^{*} \tag{19}$$

and the p-value of the test can be derived from the chi-square distribution with the degree of freedom of $N_{GC}=N_I¡ÁN_X¡Áp$.

Under suitable regularity conditions, the OLS coefficient $\widehat{\beta}_3$ obtained from Equation 15 is a consistent estimator of the actual coefficient associated with the variables being tested for Granger causality as the sample size T tends to infinity [3]. Moreover, the test statistic LM produced by Equation 16 asymptotically follows a chi-squared distribution with N_{GC} degrees of freedom, where N_{GC} equals the product of the lag order p and the number of elements in the sets of potential Granger causes and effects [3]. These properties ensure the validity of the inference results obtained after the PDS procedure.

3. Networks in Volatilities

The following analysis is based on 40 constituent stocks of the Shanghai Stock Exchange 50 Index (SSE 50). The SSE 50 comprises the largest and most liquid companies listed on the Shanghai Stock Exchange. Forty stocks are selected from this index to ensure a balance between market representativeness and computational feasibility for high-dimensional modeling.

These stocks collectively reflect the systemic dynamics of China's equity market, capturing the interactions among leading firms in sectors such as finance, energy, manufacturing, and technology. Meanwhile, the high capitalization and liquidity of SSE 50 constituents ensure the reliability and comparability of return data, which is crucial for Granger Causality Test in high-dimensional VAR systems.

Daily closing prices are applied to all selected stocks. The data is publicly available. To stabilize variance and obtain stationary series suitable for GARCH fitting, all prices are transformed into logarithmic returns before conducting the analysis. Each stock series is first fitted by a GARCH(1,1) model, and the logarithmic transformation of the conditional variance series is then used as input variables in the PDS-HDGC procedure. The sample period covers observations from January 2020 to December 2024 ($T=1212$), providing sufficient length to estimate high-dimensional VAR models and to examine the evolution of inter-stock causalities over time.

Figures below present the spillover effect networks obtained from the Figure 1 PDS-HDGC tests and Figure 2 the bivariate Granger Causality Tests. In the PDS procedure, the BIC criterion was employed for parameter tuning, with a 5% significance level. Results indicate that among all possible 1,560 causality combinations, the PDS-HDGC method, based on GARCH fitting, identified 182 significant pairs, while the bivariate Granger Causality Tests revealed 409 significant pairs. In comparison, the PDS-HDGC method identifies causality more clearly, while retaining the ability to detect weak correlations under controlled conditions for omitted variable bias.

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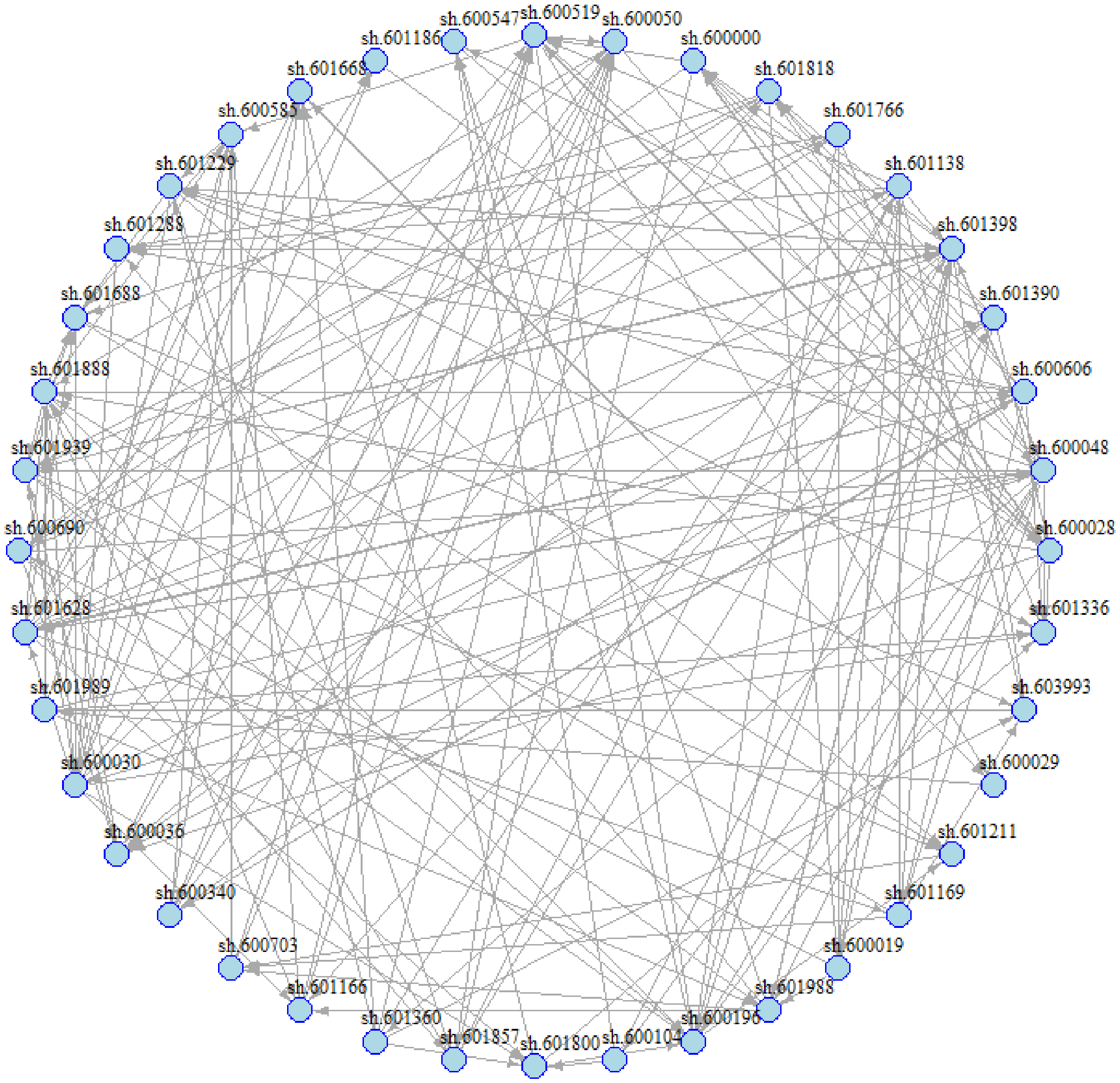

Fig. 1 PDS-HDGC

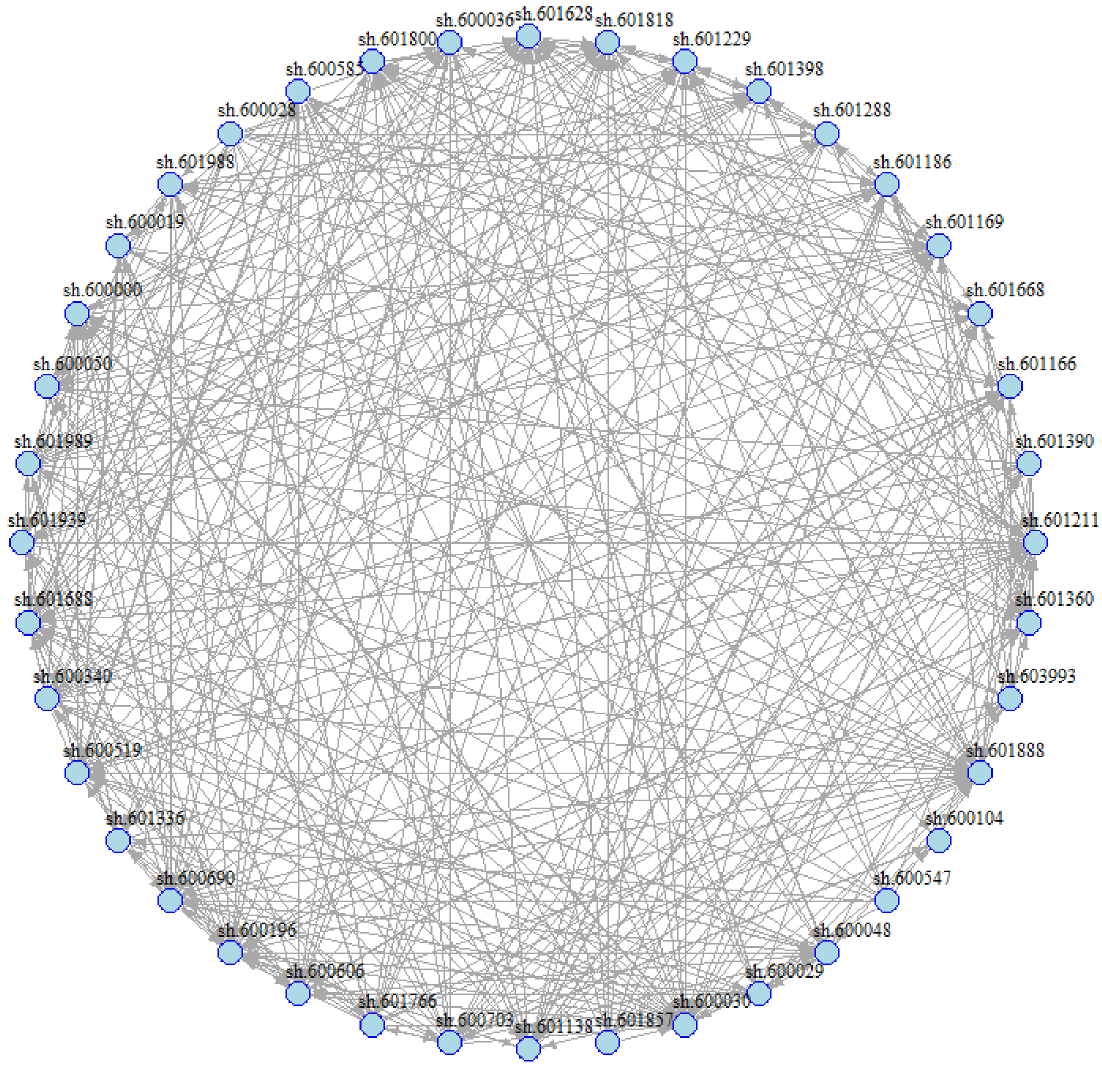

Fig. 2 Bivariate Granger Causality Tests

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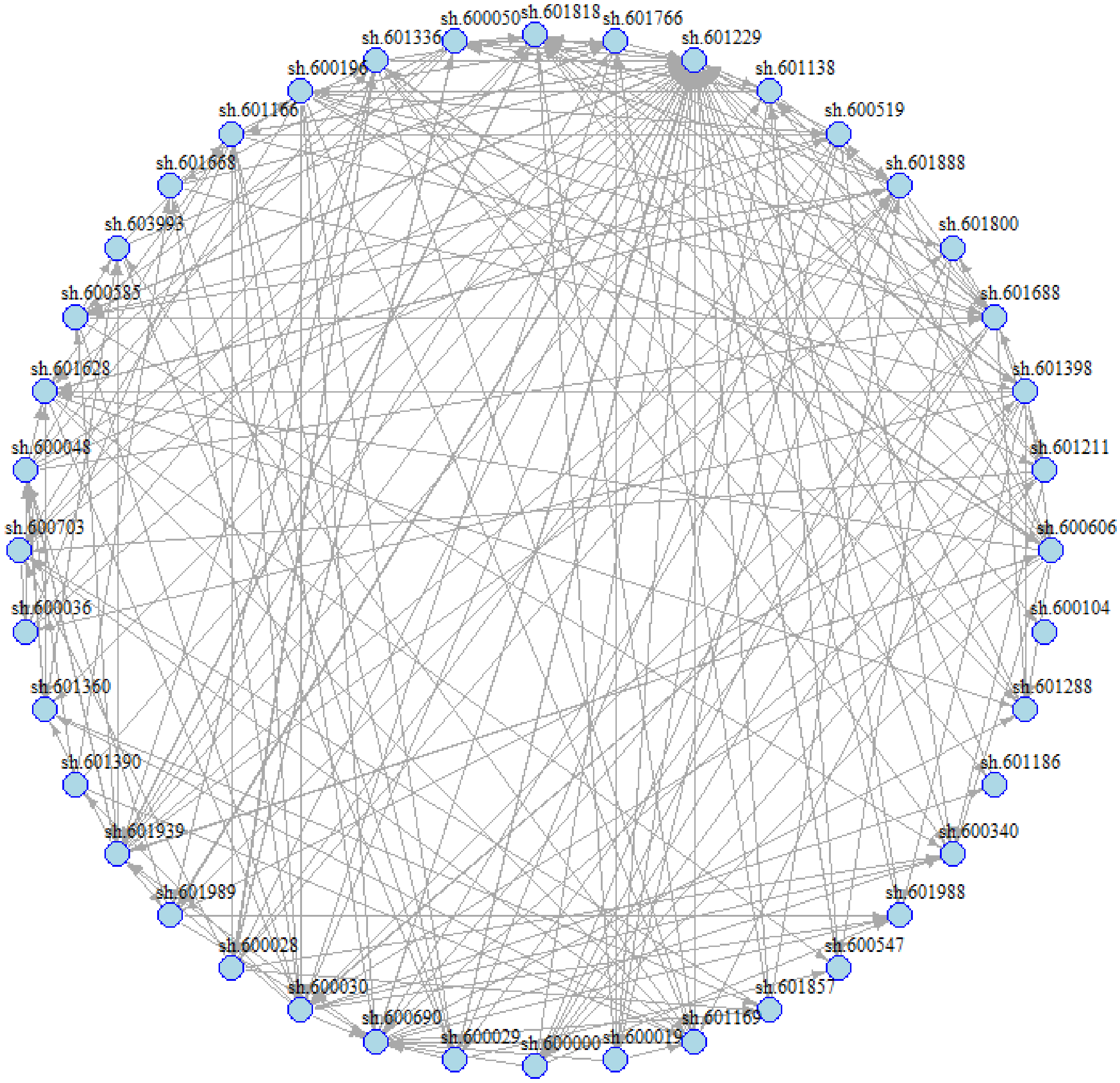

Fig. 3 Robustness test: PDS-HDGC

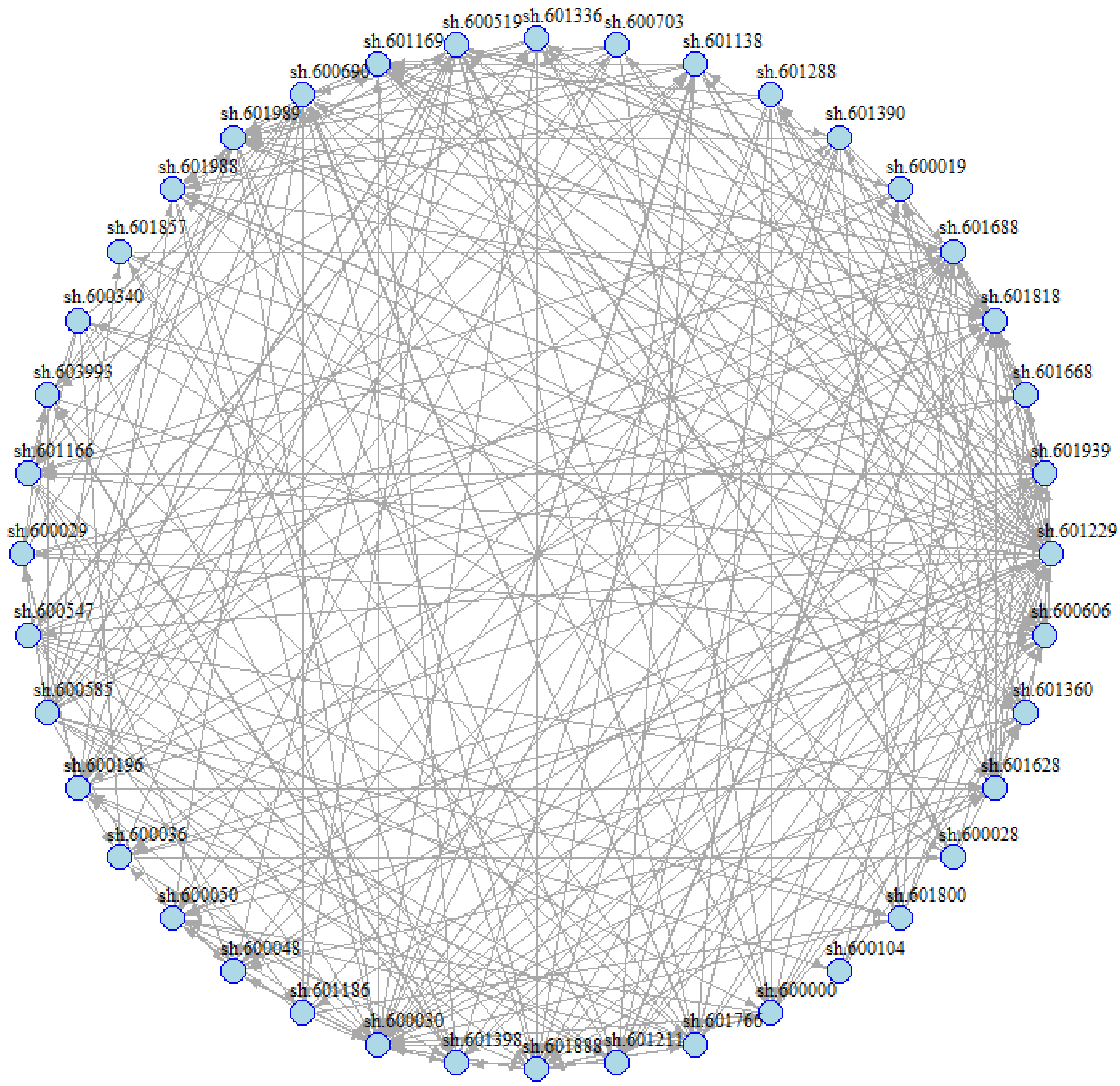

Fig. 4 Robustness test: Bivariate Granger Causality Tests

Robustness tests on a shorter period from January 2020 to December 2022 ($T = 727$) was also implemented. During this period, the COVID-19 pandemic caused significant external shocks to China's stock market, and the risk structure of the stock market may change. It also enables the study of performance differences between PDS-HDGC and the bivariate Granger Causality Test in a smaller sample size. Figure 3 and Figure 4 presents the spillover effect networks derived from the robustness tests. Among all possible 1,560 causality combinations, the PDS-HDGC method, based on GARCH fitting, identified 234 significant pairs, while the bivariate Granger Causality Tests revealed 297 significant pairs. These results demonstrate that bivariate Granger Causality Tests are more sensitive to the dimension of data compared with the PDS-HDGC method, with test results potentially exhibiting reduced consistency. In comparison, the PDS-HDGC method combined with GARCH model exhibits a certain level of robustness even on a shorter period.

4. Conclusion

The primary contribution of this article lies in combining the GARCH model with the PDS-HDGC test to examine

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the risk contagion effect of stocks at the daily level. The method proposed in this paper makes it feasible to conduct PDS-HDGC tests using daily data, allowing researchers to uncover risk transmission across stocks without relying on higher-frequency observations.

The paper first reviews the foundation of applying for the PDS in Granger Causality Test. Then the GARCH model serves as an effective volatility filter, providing well-conditioned inputs for the PDS-HDGC tests. PDS method selects control variables by performing penalized regularized regression on both the independent and dependent variables. Variables with non-zero coefficients in at least one of these two steps are retained as control variables for final testing, thereby preserving information while selecting variables.

In the empirical analysis, the PDS-HDGC test is employed to identify volatility spillover effects among forty stocks, with results compared with those from the bivariate Granger Causality Test. Findings demonstrate that the PDS-HDGC method more accurately identifies latent volatility spillover relationships between stocks as well as reduces the impact of omitted variables, thereby enhanced the practical guidance value of research results. This article also constructs a volatility influence network to visualize the test results, and robustness tests are conducted using a subsample of stock price data from 2020 to 2022.

However, the GARCH-PDS-HDGC combination method does not account for a correction of multiple testing, which may result in an inflated significance level when numerous causal relations are examined simultaneously. The purpose of the method is to provide a general picture of pairwise dependencies rather than to identify every significant causal link. Future research could be built on this limitation by developing procedures that distinguish specific causalities.

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