Dean&Francis
ISSN 2959-6157

Analysis of Common Window Function Selection Strategies for FIR Filter Design Using Window Function Methods

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Abstract:

Digital Signal Processing (DSP) technology, as a crucial discipline in communications and computer processing, has found extensive application across various fields. As a core and fundamental component in the field of digital signal processing technology, the primary function of a filter lies in the effective processing of signals. Based on the characteristics of the unit impulse response over time, filters can be categorized into two main types: finite impulse response (FIR) digital filters and infinite impulse response (IIR) digital filters. This paper first introduces the principles of FIR filters, then explains the window function method for filter design. It subsequently focuses on analyzing the characteristics and advantages/disadvantages of the five commonly used window functions in FIR filter design. By doing so, it identifies the practical applications of each window function and derives selection strategies for choosing the appropriate window function when designing FIR filters under different conditions. This approach better addresses challenges encountered in the practical design of FIR filters. At the same time, this paper looks forward to future optimizations and improvements.

Keywords: FIR filter, Window function method, Main lobe width, Side lobe level peak, Side lobe roll-off rate.

1. Introduction

Communication and computer technologies are advancing at an unprecedented pace. Consequently, the demand for DSP technology is growing steadily, leading to its remarkable development. DSP technology is applied in multiple fields[1,2]. In biomedical engineering, it is used for the extraction and processing of electrocardiogram (ECG) and electroencephalogram (EEG) signals, and also involves the processing of certain medical images. In audio signal processing, the implementation of graphic equalizers is frequently encountered. Within control systems, DSP is employed for signal conditioning and real-time feedback. In the field of image processing, image denoising and compression also require and utilize DSP technology [3]. In cutting-edge fields such as artificial intelligence, DSP technology also plays a significant role. It has an indispensable impact in areas including AI/deep learning-based filter design (using neural networks to automatically design filter coefficients), reconfigurable filters, and efficient hardware implementations based on FPGAs [4]. In digital signal processing technology, the digital filter is a core and critical component[5]. Its primary function lies in effectively processing signals—specifically, filtering and extracting relevant information. Therefore, its design poses a significant challenge within the field of digital signal processing. Based on the characteristics of the unit impulse response over time, filters can be broadly categorized into two main types: FIR digital filters and IIR digital filters. The unit impulse response of an FIR filter is finite in duration. Its output depends on both the current and past input signals, but not on the output signal itself. Therefore, it exhibits strict linear phase. Signals processed through it incur no phase distortion. The unit impulse response of an IIR filter is infinite in duration. Its output depends on the current and past input signals, as well as the entire history of input signals. It is suitable for scenarios requiring precise phase preservation, such as audio processing. When implementing high-selectivity filtering using FIR filters, a high-order filter is typically required, resulting in substantial computational demands and significant hardware resource consumption. IIR filters, owing to their highly efficient frequency-selective characteristics, require significantly less computation than FIR filters to achieve the same functionality. Consequently, they are well-suited for applications demanding high real-time performance. Because its output is influenced by both current and past input signals as well as historical output values, IIR can only achieve approximate linear phase. There are three common methods for designing FIR filters: the window function method, the frequency sampling method, and the optimal ripple approximation method. Among these, the window function method can be further subdivided into six distinct approaches based on the use of different window functions for filter design. This paper will examine the characteristics and advantages/disadvantages of these six window functions based on their respective features, thereby deriving a selection strategy for choosing window functions according to actual signal processing requirements to guide practical applications.

2. Principles of FIR Filters

A FIR digital filter is essentially a linear time-invariant discrete system implemented using finite-precision algorithms. It exhibits causality, meaning its impulse response is zero prior to the application of an impulse signal. Upon receiving a finite-length discrete sequence, it outputs a sequence that is distortion-free and time-shiftable [4]. It ensures that the amplitude characteristics meet technical requirements. Furthermore, as long as the coefficient sequence h[n] of the FIR filter satisfies a certain symmetry (such as even symmetry or odd symmetry about the center point), it exhibits strict linear phase. Its transfer function H(z) is given by Equation (1):

$$H(Z) = \sum_{n=0}^{N-1} h(n) z^{-n} \tag{1}$$

Here, h(n) denotes the impulse response coefficient of the filter, n represents the discrete-time index, N indicates the order of the filter, and z^(-n) signifies the negative power of z, representing a delay operation in the z-domain. H(z) is the N−1th-degree polynomial in z−1. It has N−1 zeros in the complex plane and an N−1-fold pole at the origin z=0. Therefore, H(z) is absolutely stable. The output y(n) of an FIR filter is the convolution of the input signal x(n) with the filter coefficients h(m), as shown in Equation (2):

$$y(n) = \sum_{m=0}^{N-1} h(m) x(n-m) \tag{2}$$

Here, n represents the time sample; N denotes the number of coefficients; m is the current sample index used to traverse all coefficients.

3. Window Functions

3.1 Principle of Window Functions Page Numbers

The fundamental principle of the window function method is to find a function H(ejw) that approximates the desired ideal filter response function Hd (e jw) [6]. From the perspective of the unit impulse response sequence, the goal is to make the designed filter's h(n) approach the ideal unit impulse response sequence hd(n) [7]. By performing an inverse Fourier transform on the desired frequency response (such as an ideal low-pass or high-pass filter), the corresponding unit impulse response is obtained. This response is then truncated using a finite-length window function w(n), yielding the desired unit impulse response h(n) = w(n)hd(n) [8]. h(n) is the unit impulse response of a causal FIR filter that approximates the ideal frequency response.

3.2 Window Function Categories

Common window functions include rectangular windows, triangular windows, Hann windows, Hamming windows, Blackman windows, and Kaiser windows [9]. Key parameters requiring special attention are listed in Table 1.

Table 1. Explanation and Characteristics of Core Window Function Parameters

Core Parameters	Explanation	Characteristics
Main Beam Width	Peak width of the main lobe at 3 dB (cutoff frequency)	The narrower the main lobe width, the higher the frequency resolution.
Side lobe peak level (dB)	The ratio of the amplitude of the highest side lobe to the amplitude of the main lobe	The lower the side lobe level, the lower the degree of spectral leakage.
Side lobe roll-off rate (dB/octave)	The rate at which the side lobe level decreases as frequency moves away from the main lobe.	The faster the roll-off rate, the better the interference suppression.

3.2.1 Rectangular Window

In the time domain, a window function is a finite-length weighted function. The time-domain expression for a rectangular window is:

$$w(n)=1, n=0,1,...,N-1 \quad (3)$$

Here, N represents the length of the time-domain waveform. It can be seen that this value remains unchanged regardless of variations in the time variable. This is equivalent to directly truncating the signal. Its main lobe width is the narrowest among all window functions (4π/N) [10], enabling it to provide the highest frequency resolution. However, its side lobe level peak (-13 dB) is relatively high, resulting in greater leakage compared to other window functions. Its roll-off rate (-6 dB/octave) is slow, leading to weaker suppression of interference signals. Therefore, rectangular windows are suitable for applications requiring high frequency resolution but without strict requirements for leakage or interference resistance.

3.2.2 Triangle Window

The discrete-time domain expression for the triangular window is:

$$w(n)=1-\frac{2\left|n-\frac{N-1}{2}\right|}{N-1}, n=0,1,...,N-1 \quad (4)$$

It is formed by convolving two rectangular windows. Its main lobe width (8π/N) is twice that of a rectangular window, resulting in poorer frequency resolution compared to the rectangular window. However, its peak side lobe level (-27 dB) and side lobe roll-off rate (-12 dB/octave) are lower than those of the rectangular window, thereby enhancing frequency leakage control and interference immunity. Compared to rectangular windows, it offers superior frequency leakage and interference resistance, but this comes at the cost of inferior frequency resolution. Its time-domain waveform is an isosceles triangle.

3.2.3 Hanning Window

The time-domain expression for the Hanning window is:

$$w(n)=0.5-0.5cos(\frac{2\pi n}{N-1}) \quad (5)$$

The Hanning window is a raised cosine window, featuring a smooth transition to zero at both ends of its waveform with a raised peak in the middle. Its side lobe level peaks are relatively low (-31dB) and exhibit rapid attenuation (-18dB/octave), resulting in minimal frequency leakage and strong resistance to interference. However, its main lobe width (8π/N) is twice that of a rectangular window, resulting in a lower resolution compared to a rectangular window. Hanning windows strike a relatively ideal balance between frequency resolution and interference resistance, making them well-suited for signal processing in the audio spectrum and thus relatively widely used.

3.2.4 Hamming window

The time-domain expression for the Hamming window is:

$$w(n)=0.54-0.46cos(\frac{2\pi n}{N-1}) \quad (6)$$

Hamming windows are an improvement over Hanning windows. Compared to the Hanning window, it reduces the maximum sidelobe level (-43dB), resulting in lower leakage. This makes it suitable for applications demanding stricter frequency leakage control, such as voice signal processing. However, its side lobe roll-off slows (-6 dB/octave). Consequently, its interference resistance is relatively lower than that of the Hann window.

3.2.5 Blackman Window

The time-domain expression for the Blackman window is:

$$w(n)=0.42-0.5cos(\frac{2\pi n}{N-1})+0.08cos(\frac{4\pi n}{N-1}) \quad (7)$$

The Blackman window is a second-order cosine window, which adds an extra harmonic term to the cosine window to make the window function smoother. Its side lobe level peak (-58 dB) is extremely low, and the side lobe roll-off rate is exceptionally steep (-18 dB/octave). Consequently, it exhibits outstanding interference resistance and minimal frequency leakage. However, a significant drawback is that the main lobe width (12π/N) is three times that of a rectangular window, resulting in relatively poor frequency resolution. It is primarily used in situations where weak signals need to be detected or captured.

3.2.6 Kaiser Window

The time-domain expression for the Kaiser window is as follows:

$$w(n)=\frac{I_0\left(\beta\sqrt{1-(\frac{2n}{N-1}-1)^2}\right)}{I_0},n=0,1,...,N-1 \qquad (8)$$

β is the shape parameter, which determines the shape and performance of the window. As β increases, the time-domain waveform of the window will become smoother at both ends, while in the frequency domain, the side lobe levels decrease and the main lobe width increases. I_0() is the first-kind zero-order modified Bessel function, which enables the Kaiser window to achieve the lowest side lobe level at a given main lobe width, or the narrowest main lobe at a given side lobe level. By designing the β parameter, the main lobe width and peak side lobe level of the window function can be adjusted at any time. This allows for tailored design based on actual requirements regarding frequency resolution and leakage, resulting in the most appropriate window function. However, this comes with a massive computational burden, a drawback that Kaiser Windows must take into account.

Table 2 summarizes the core parameters and applicability conditions of the five window functions mentioned earlier for reference.

Table 2. Summary of Parameters for Five Window Functions

Window function	Main petal width	Side lobe peak level (dB)	Side lobe roll-off rate (dB/octave)	Applicable Conditions
Rectangular window	4π/N	-13	-6	Precise frequency measurement
Triangular window	8π/N	-27	-12	Theoretical Analysis
Hanning window	8π/N	-31	-18	Audio Processing
Hamming window	8π/N	-43	-6	Speech Processing
Blackman window	12π/N	-58	-18	Detect weak signals

Table 3 summarizes the approximate window functions corresponding to different beta coefficients for Kaiser windows.

Table 3. Window Functions Approximated by Different Beta Coefficients for Kaiser Windows

β coefficient	Window Functions (Approximation)
β=0	Rectangular window
β=5	Hamming Window
β=6	Hanning Window
β=8.6	Blackman Window

4. Comparative Strategy Analysis

Ideally, the narrower the main lobe width of a window function, the more precise its frequency resolution, the faster its side lobe roll-off rate, and the stronger its ability to filter out interference signals and resist interference. The lower the side lobe level peak, the less frequency leakage occurs. Generally, it is desirable to achieve the narrowest possible main lobe width and the steepest possible side lobe roll-off. However, as the main lobe width decreases, the side lobe level peaks tend to increase, and the side lobe roll-off rate slows down. Therefore, in practical design, it is necessary to balance frequency resolution, interference resistance, and leakage levels based on different requirements to determine which window function to use for the design. As analyzed above, when the window length N is the same, filters designed with rectangular window functions exhibit the narrowest main lobe width and the narrowest transition bandwidth. However, it exhibits the poorest stopband attenuation, resulting in the most precise frequency resolution, while simultaneously offering the poorest interference resistance and leakage performance metrics. Therefore, rectangular windows are suitable for applications requiring high frequency resolution while not demanding strict leakage control. For instance, when analyzing transient signals whose duration equals the window length, they can be directly captured using a rectangular window or subjected to precise ampli-

ISSN 2959-6157

tude measurements for sine waves of known frequency. The filter designed using the Blackman window function exhibits the widest main lobe width and the broadest transition bandwidth. However, it features the narrowest stopband and the highest stopband attenuation, resulting in the lowest frequency resolution while delivering optimal interference immunity and leakage performance. Suitable for applications with extremely high requirements for leakage current and anti-interference capability, such as detecting weak signals or in calibration and testing scenarios (e.g., ADC performance testing). As a cosine window, the Hann window achieves a relative balance between the main lobe width and the side lobe peak level, effectively balancing three key performance metrics: frequency resolution, interference resistance, and leakage. This approach is suitable for applications requiring both frequency resolution and leakage suppression, such as audio and vibration analysis, or identifying the frequency components of position signals. The Hamming window exhibits lower peak levels in the side lobes compared to the Haning window, making it suitable for applications with stricter leakage requirements. For instance, the Hamming window is more appropriate for voice processing. Kaiser windows are suitable for applications with complex and variable signal processing requirements because their performance can be adjusted at any time via β parameters.

Table 4 summarizes the selection strategies.

Table 4. Summary of Window Function Selection Strategies

Window function	Characteristics	Selection Scenarios
Rectangular window	The most precise frequency resolution, but the worst interference resistance and leakage performance.	1. When analyzing transient signals, their duration equals the window length. 2. Perform precise amplitude measurements on sinusoidal waves of known frequency.
Hanning Window	Balances frequency resolution, interference immunity, and leakage level across these three- performance metrics.	1. Audio and Vibration Analysis 2. Frequency Components of Location Signals
Hamming Window	Compared to the Hann window, it exhibits lower peak levels in the side lobes.	Speech Processing
Blackman Window	Lowest frequency resolution, optimal interference resistance and leakage performance	1. Detecting weak signals 2. Calibration and testing (e.g., ADC performance testing)
Kaiser Windows	Performance adjustments can be made at any time via β parameters.	In scenarios where signal processing requirements are complex and variable

5. Conclusion

This paper first briefly summarizes the significance of FIR filters and the principles of designing FIR filters using window functions. It then analyzes the characteristics of five distinct categories of window functions, thereby determining the applicability of each in practical scenarios. Finally, it summarizes and recommends selection strategies. The core issue in selecting window functions lies in the trade-off between main lobe width and side lobe level peaks. The narrower the main lobe width, the better the frequency resolution; the lower the side lobe level peak, the less frequency leakage. However, narrower main lobes also result in higher peak levels in the side lobes. Therefore, selecting an appropriate window function based on the actual processing requirements of the signal is crucial. Looking ahead, we could explore integrating artificial intelligence to develop a multifunctional filter capable of supporting multiple window functions.This filter would leverage AI algorithms to automatically select the optimal window function based on processing requirements and the characteristics of the signal being processed. The efficiency and capability of this filter in processing actual signals will be significantly enhanced.

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