

Application of Copula Vector Autoregression Model in Multi Asset Portfolio Risk Management

Bolun Zhang^{1,*}

¹School of Statistics and
Mathematics, Henan Finance
University, Zhengzhou, China

*Corresponding author:
zhangbolun599@gmail.com

Abstract:

It is challenging to use the covariance models and single VAR models to capture these complicated features accurately, and it may result in biases of risk evaluation. Nevertheless, the Copula model is capable of efficiently describing multidimensional dependency structures. The dynamic interactions among the time series can be effectively characterized by Vector Autoregression (VAR) model. The consolidated Copula VAR model is not only able to reflect effectively the complex dependence relationships between assets in a multiple asset portfolio, but also flexibly capture the transmission and diffusion process of risk, and as such enhances goal accuracy of risk prediction as well as work efficiency of management. Firstly, this paper is organized as: in the second section introduces the theory of Copula model and VAR model, these two theoretical, current application status in financial risk management, and its integration necessity and advantages were analyzed. From the empirical results, build a Copula VAR model and test its performance of multi asset portfolio risk assessment by concentrating on how well the model is capable of capturing risk in extreme market circumstances. Compared with the traditional risk management methods, Copula VAR model proved to be better on risk finding out, risk transmission path analysis and risk early-warning.

Keywords: Copula model; Vector auto-regression (VAR); multi asset portfolio; risk management

1. Introduction

Traditional risk management approaches primarily depend on linear correlation coefficients and historical volatility for gauging risk relation among assets

but are severely limited in multi asset portfolios. It is well known that the linear correlation can not reflect the nonlinear dependence and tail dependence among assets, which leads to serious underestimation of risk when the extreme market occurs. Classical models

often assume the distribution of asset returns is a normal one, but this approach ignores the fat tails and skewness found in financial markets by nature and subsequently lead to an underestimation of risk. The static risk model is not easy to reflect the dependent relationship of dynamic changes by various assets, and can not detect in time the splash effect caused by risk contagion or volatility aggregation and so on, which affecting the timely warning of risks.

The relationships between various asset classes are intricate, and traditional linear correlation analysis is unable to adequately depict the nonlinearity and tail dependence among assets. Given the uncertainties of the risk prediction, linear correlation analysis must take into account the volatility changes and asset price dynamics resulting from some determinants that led to turbulent periods. The high frequency of extreme market events has raised the possibility that the portfolio encounters systemic risk and enhanced the complexity of risk management. How to accurately identify and measure the intricate dependent relationships among multi-assets, dynamically reflect the risk transmission process, and be able to improve the level of asset portfolio's risk management has become a focus in recent years.

This study wants to establish and verify Copula - VAR mixed model by combining Copula with vector auto-regression (VAR) to enhance the accuracy for risk measurement and forecasting. The study seeks to overcome the limitations of traditional correlation coefficients by examining the complex and nonlinear dependence structures among assets within a diversified portfolio. Applying VAR model to describe the dynamic transmission mechanism of asset returns and extract the propagation path of risk spillover between different assets. Empirical test the risk measurement of Copula VAR model during extreme market volatility. Recommend practical and operational risk control strategies to investors for the purpose of investment asset allocation optimization according to the model results. Through the above research, it is hoped that this paper can offer theoretical reference and practical suggestions for multi asset portfolio risk management, assist financial institutions to better manage market risk, and also encourage innovative development of risk management technology.

And the key research problem at present is how to measure the complex dependence relationship and dynamic changes among assets in multi asset portfolio risk management in a reasonable way. The conventional approaches are not able to fit non-linear and tail dependence characteristics well with heavier reliance nonlinear correlation coefficients, which leads risk estimation into biased manner. On the basis of this, three important problems will be

addressed in the paper: 1) How to build one model framework which is expressive enough to capture non-linear interference structure among multiple assets. Secondly, how to integrate dynamic time-series information to trace risk transmission pathways and their time-varying properties. How to make the model more robust and predicting in harsh market. How to transition from the model results to real world asset allocation and controlling risk, plus how the portfolio can offer improved returns after adjusting for risk. To this end, carry out empirical tests on these models that systematically consider the performance of Copula VAR under different stages of the market, with a view to assess their detection capabilities for risk contagion mechanisms and warning efficiency to provide theoretical support and practical guidance for managing risks in multi asset portfolios.

The traditional models put much emphasis on the linear correlation, and thus it is very hard to grasp complex nonlinear interrelations and tail risks among assets. The Copula VAR model overcomes this impasse by blending the powerful description for multidimensional dependent structures facilitated by Copula functions together with the perfect measurement of varying time delayed interactions provided by VAR models, making significant improvements over accuracy and completeness in measuring risk. It has been shown that this model can much better show the asymmetric dependencies between assets and the risk contagion paths under extreme events, as well as provide a more scientific basis for managing risks of multi-asset portfolios.

The linear correlation coefficient used in traditional method cannot describe complex non-linear relationships between assets, which leads risk is unseen. The Copula VAR model is a creative method which integrates the advantage of characterizing the dependency structure flexibly based on Copula functions and capturing dynamical temporal linkages using VAR model, thus it can more effectively describe risk transmission mechanism among assets to enhance sensitivity and timeliness of risk warning. When the model is applied, it find stronger ability in identifying risk when markets suffer serious fluctuations, which can help investors to adjust asset allocation immediately and reduce possible lost.

2. Current Research Status in China

2.1 Research Status of Multi Asset Portfolio Risk Management

Fortunately, domestic scholars have achieved some results in the acceptance of multi asset portfolio risk management, most of which are for new method of risk measure-

ment and model further application innovation. Xu Gang et al. proposed the Copula model that expanded the scope of linear correlation, accurately described complex interdependency relations of assets and enhanced VaR prediction accuracy on stock market among multi asset combination [1]. Zhang Chao applied the hybrid Copula model to financial leasing risk management and proved it can be used in multi asset risk identification. Researchers analyzed the extreme risk contagion mechanism of crude oil, gold and US dollar combinatorial by Copula VaR model in depth to reveal the ability of Copula VaR model to capture non-linear dependence and tail risk [2,3]. Chen Rui has developed the Vinecopula method to enhance the VaR predicting accuracy of multi asset portfolios [4]. Zhang Yi studied the application of Copula function and VaR model in Public-Private Partnership (PPP) project financing risk, which showed that the method has been widely used [5]. In general, the domestic study is developing toward the combination of multivariate models and making more accurate characterization of dependency structure and dynamic analysis of risk transfer.

2.2 Exploration of the Application of Copula Model and VAR Model in China

Copula-VAR Models in Financial Risk Management In recent years, domestic scholars have vigorously explored the Copula and VAR models and their practical applications in this field. Hou Rong applied the time-varying Vine Copula model to the deep mining of nonlinear and tail-dependent properties in asset portfolios, which greatly improved the accuracy of risk measurement [6]. Its study offers a more direct approach to analyze the dependence structure of high-dimensional financial data, and can facilitate wider application of copula theory in asset pricing and risk management. Shen Yongao et al. focused on the significant role of dynamic forecast of risk, e.g VAR model with VaR technique and Garch model [7]. Yan Xinyue carried out a detailed study on the application of VaR model in securities investment, and pointed out the deficiency of traditional methods, and proposed to apply skills of flexible dependence structure modals to enrich risk identification [8]. These studies show that domestic researchers have identified the entitative value of two merged models and have attempted to address the issues of complex dependence and dynamic conduction in multi asset portfolios. Nevertheless, previous studies have paid limited attention to the discussion of theoretical construction and empirical evidence is relatively small in scale. At the same time, future work should also enhance the research of model practicality and applicability so as to solve the problem of advanced risk management in a com-

plex financial environment.

2.3 Shortcomings and Development Trends of Copula VAR Model Related Research

Although the Copula model and VAR model have achieved significant results in the field of risk management, research on the Copula VAR model that integrates the two is still insufficient. Ying Chengyi pointed out that traditional risk management methods are difficult to effectively capture the complex interdependence between assets, and there is an urgent need for more flexible models [9]. Although Diao Fei made some breakthroughs in the long memory mixing Generalized Autoregressive Score (GAS) Copula model, he did not fully integrate the dynamic characteristics of the VAR model [10]. Chen Baiyu enhanced tail risk capture through the Generalized Autoregressive Conditional Heteroskedasticity - Extreme Value Theory (GARCH-EVT) copula model, but there are still shortcomings in modeling dynamic causal relationships [11]. Sun Yihan pointed out that existing models are difficult to comprehensively identify market nonlinear dependencies and risk contagion pathways [12]. Future research should deepen the integration of Copula and VAR, enhance the ability to characterize the nonlinear dependencies and dynamic risk transmission of multi asset portfolios, in order to better address the challenges of risk management in complex financial environments.

3. Current Research Status Abroad

3.1 Progress in the Application of Copula Model in Financial Risk Management

The Copula model has become one of the core tools driving the development of financial risk management technology due to its excellent flexibility in characterizing nonlinear and symmetric dependency relationships of variables. Its applications have covered multiple fields such as market risk and credit risk. Its model has received widespread attention and in-depth application in the field of financial risk management. Janabi et al. used the vine Copula method to optimize the liquidity adjusted value at risk (VaR) of multi asset portfolios, effectively improving the accuracy of risk measurement, especially in capturing complex dependencies between assets [13]. Madhusudan and Samit combined conditional heteroskedasticity model (CGARCH), extreme value theory (EVT), and Copula method to construct a portfolio risk management framework suitable for high-frequency trading environments, significantly improving the risk prediction performance of VaR and CVaR, especially showing strong adaptability

during extreme market volatility periods [14]. The Copula model provides a more accurate and dynamic tool for financial risk management by flexibly characterizing the nonlinear dependencies between assets, promoting continuous innovation and improvement of risk measurement methods.

3.2 VAR Model and Its Development in Multi Asset Risk Analysis

The Vector Auto regression (VAR) model, which is of great utilization in multivariate time series analysis, is also commonly used for multi series risk analysis since it has the advantage to capture coefficient restrictions and dynamic causality among variables. Makarov Pointed out that the VAR model of jump diffusion process based on systemic risk factors can more realistically describe asset price fluctuations and risk transmission mechanism, greatly promote the accuracy and practicability in quantitative risk management [15]. Flores and Díaz-Hernández stress that VAR models model nonlinear dependence structural tail dependence, which is very important for the risk management of multi asset portfolios in extreme market conditions [16]. The establishment of VAR model in multi asset risk analysis not only improves theoretical model system, but also helps understanding and predicting the dynamic evolution of market complex risk environment by collocating with other models.

3.3 Research and Case Analysis on the Fusion of Copula VAR Model

Researchers have paid more and more attention to the combination of Copula model and VAR model in order to better capture the complex dependence relationships as well as dynamic risk transmission mechanisms cross multiple asset portfolios. Liangxiong and Muhammed creatively combined deep learning and Markov models to enhance the performance of the Copula method in risk measurement, demonstrating its capability to capture nonlinear dependence structures [17]. Agenor et al. initiate with economic and financial risks, Bruneau et al. Utilized Copula function to research the risk sensitive of multi-asset portfolio, and combined VAR mode to analyze the propagation path of risk among asset, they proved that the fusion model was effective in practice risk management [18]. These studies not only enrich the theoretical system of multi asset risk management, but it also provides a good technical means for the risk control in practical investment portfolio.

4. Conclusion

This study demonstrates that the Copula-VAR model, by leveraging the Copula function's strength in characterizing multidimensional dependence structures and the VAR model's capability in capturing dynamic time-series relationships, can better reflect the complex dynamic mechanism of risk transmission among assets in a multi-asset portfolio. Empirical analysis of the model has proved its good performance in identifying risk aggregation and transmission paths under extreme market conditions, which also effectively enhances the precision of risk prediction and early warning. The Copula VAR and multi asset portfolio risk management As opposed to the classic risk management methods, the Classic VAR model is improved by the Copula VAR model and its nonlinear dependency relationship is better captured in traditional assets, which resulted in more scientific and effective risk control of multi asset portfolios. The Copula VAR model proposes a new technical method for risk management of multi-asset portfolio, full of theoretical value and wide application prospects, bringing great support to the investors risk identification and control in complex market conditions.

Multi asset portfolio risk management is becoming a challenging task with the rapid evolution of world financial markets and the diversity of assets. In what follows, this paper anticipates that the Copula VAR-Model will yield some insights and progress in various regards. Leveraging big data and artificial intelligence techniques, models will be more accurate and effective in capturing high-dimensional dependent relationships and the dynamic transmission mechanism of risk. The adaptability and stability of the model should be improved, so that the sudden changes of market environment or risk event can be effectively controlled to make the risk early warning more timely and accurate. The study on cross market and cross asset class risk contagion will gradually become the concern, which also encourages models to be applied in the global portfolio management process. The use of the research conclusion behind behavioral finance theory and market micro structure studies, Copula VAR model will be to quantify the influence of investor behavior on risk transmission together which could contribute to construct a more scientific and perfect regulation system of financial risk. It is hoped that future studies will pay close attention to the combination of theory innovation and practice application to boost a new level of intelligent and refined risk management in multi-asset portfolio.

References

- [1] Xu G, Cai X P, & Xiong S M. Application of Teng Copula Model in VaR Prediction of Multi Asset Portfolio in China's Stock Market. *Journal of Jingtangshan University (Natural Science Edition)*, 2020, 41(02): 8-15.
- [2] Zhang C. Application of Hybrid COPULA Model in Financial Leasing Risk Management. *Fortune Times*, 2021, (02): 117-118.
- [3] Xing K K, & Hong Z M. Risk measurement of asset portfolios of crude oil, gold, and US dollars based on Copula VaR model. *Journal of Chifeng University (Natural Science Edition)*, 2021, 37(02): 50-56.
- [4] Chen R. Based on Vine Copula model and multi asset portfolio VaR prediction. *Modern Business*, 2022, (17): 153-156.
- [5] Zhang Y. Research on the Application of Copula Function and VaR Model in PPP Project Financing Risk Measurement. *Economic Research Guide*, 2020, (28): 43-44.
- [6] Hou R. Research on Portfolio Risk Based on Time Varying Vine Copula Model. *Central University for Nationalities*, 2020.
- [7] Shen Y G. Research on the Application of VaR GARCH Volatility Improvement Model in Risk Management. *Shandong University of Finance and Economics*, 2020.
- [8] Yan X Y. VaR Model and Its Application in Securities Investment Management. *China Business Review*, 2020, (02): 36-37.
- [9] Ying C W. The Application of VaR in Credit Risk Management of Commercial Banks. *Modern Business*, 2020, (05): 111-112.
- [10] Diao F. Long memory mixing GAS Copula model and its application in investment portfolios. *Zhejiang University of Finance and Economics*, 2023.
- [11] Chen B Y. Research on Risk Management of Stock Index Fund Portfolio Based on GARCH-EVT Copula Model. *Hunan University*, 2020.
- [12] Sun Y H. Research on Risk Management of Domestic Index Investment Portfolio. *Nanjing University*, 2021.
- [13] Janabi A A M, Ferrer R, & Shahzad H J S. Liquidity-adjusted value-at-risk optimization of a multi-asset portfolio using a vine copula approach. *Physica A: Statistical Mechanics and its Applications*, 2019, 536: 122579.
- [14] Madhusudan K, & Samit P. Intraday portfolio risk management using VaR and CVaR: A CGARCH-EVT-Copula approach. *International Journal of Forecasting*, 2021, 37(3): 1326.
- [15] Makarov N R. Option Pricing and Portfolio Optimization under a Multi-Asset Jump-Diffusion Model with Systemic Risk. *Risks*, 2023, 11(12): 217.
- [16] Flores S Y, & Díaz-Hernández A. Counterdiagonal/nonpositive tail dependence in Vine copula constructions: application to portfolio management. *Statistical Methods & Applications*, 2020: 1-33.
- [17] Liang X L, & Muhamed B M. Adoption of deep learning Markov model combined with copula function in portfolio risk measurement. *Applied Mathematics and Nonlinear Sciences*, 2021, 7(1): 901-916.
- [18] Bruneau C, Flageollet A, & Peng Z. Economic and financial risk factors, copula dependence and risk sensitivity of large multi-asset class portfolios. *Annals of Operations Research*, 2020, 284(2): 165-197