

Mathematical Models in Understanding and Managing Coral Bleaching

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Abstract:

Because of the drastic changes in the climate in recent years, coral bleaching is a major threat worldwide. Mathematical models have the benefit of providing people with more potent tools to experiment with more complex environmental concerns. Dynamic model and mechanistic model are two important models of mathematical frameworks. On one side, dynamic models provide a macro-scale view of the reef ecosystems: mimic the holistic evolution of coral systems through capturing feedback loops and identifying ecological thresholds. On the other side, mechanistic models focus on elucidating the fine-scale driving of the whole process: dissect the underlying micro-physiological processes, like photosynthetic dysfunction and oxidative stress. These mathematical systems do not just amass data and offer a straightforward calculation tool, but they also allow predicting the future of uncertain situations reasonably in various climatic conditions. Moreover, the combination of multidisciplinary knowledge and quantifiable measurements indicates that such mathematical models provide potent support to conservation practices of corals and can streamline the process of evidence-based management practices. This article not only reviews how math models played roles in elucidating the causes of bleaching but emphasize how they enhance warning systems and inform the formulation of targeted conservation strategies through case studies.

Keywords: Coral Bleaching, Math Model, Dynamic Model, Mechanistic Model

1. Introduction

Coral reefs are significant to the coastal systems as they enhance the economy and sustain livelihoods. They also support fisheries and the entire coastline. But today, the coral reefs are beginning to bleach

because of the current climate change. According to the report, coral reefs have experienced severe degradation between 2009 and 2018, losing 14% of the global coral population due to coral bleaching[1]. The coral bleaching is simply the destruction of the symbiotic association between corals and their zo-

oxanthellae [2]. In the case of the abnormally high temperatures in seawater, the photosynthetic apparatus of the algae will be damaged, and hence, it will cause excess production of toxic reactive oxygen species (ROS) [3]. When the toxins are released into the coral, the host starts to expel the alkaloids as a self-defense mechanism.

Being one of the hotspots of recent environmental issues, the research on protecting the coral reefs is extensive. Traditional experimental observation and statistical techniques are only capable of showing correlations and not the underlying dynamical processes and causal feedback loops, since it is a multidisciplinary and nonlinear process, making employment of physiology, ecology, and environmental sciences topics. Therefore, the math model will be an invaluable analytical tool, which can strongly support its approach in the consideration and solution of coral bleaching. Various math models, such as dynamic modeling and mechanistic differential equation models, are used by scientists to approach the problem in various directions. As an example, Antonelli et al. used a nonlinear model to discuss the effect of thermal stress on breaking the stability of the coral-zooxanthella symbiosis, which demonstrates that mathematical models can mirror concepts in complex systems that cannot be easily found using intuition or observed experiments [4]. It is also more realistic with longer interactive models with multi-species interactions by Barrozo et al that combine the effects of disease and algal competition on coral bleaching and recovery [5]. Moreover, Boonnam et al. have experimented with it, combining predictive modelling and machine learning to show new opportunities in predicting bleaching more accurately in climate change circumstances. In this article, the author notes that this mathematical model has excellent predictive abilities, whereby the accuracy rate is up to 91% and the AUC value is up to 0.94[6]. It has also enhanced the effectiveness of early warning by about 15 percent, and the false alarm rate has been cut short by a substantial margin over traditional methods [6]. Accordingly, this study aims to systematically elucidate how mathematical models resolve the coral bleaching problem in novel aspects by reviewing and analyzing case studies. The review will focus on three dimensions: First, separate analysis of fundamental principles of dynamic and mechanistic models. Second, the application of two models in case studies especially the predictions of climate driven bleaching events. Third, a discussion on the adaptability and practical value of these models in informing early warning systems and government conservation strategies. By integrating insights of different kinds of typical cases, this review highlight how such such systematic model deepen the mechanistic understanding of bleaching process and provide quantitative support for science-based

management decisions.

2. Dynamic Modeling

2.1 Dynamic Modeling's Principles

Dynamic modeling is a method of studying the evolutionary processes of complicated systems over time. It focused on how variables impact each other and how they changed over time, and their long-term outcome, instead of just the static correlation between variables. Also, the dynamic model treats an organization as a complex system with multiple interrelated elements, and these elements form a feedback loop: a change in one variable can, in turn, affect itself through a series of intermediate variables. Three main functions of the dynamic model are causal loop diagrams (CLDs), stock and flow diagrams (SFDs), and system dynamics diagrams [7]. CLD is a qualitative model that can visually demonstrate causal relationships and feedback loops between variables. SFDs, then, convert CLDs into a computational structure, which is quantifiable. Lastly, according to SFDs, a system dynamics diagram is a mathematical model that subsequently simulates the behavioural patterns of systems given different conditions and policy interventions as time progresses [8].

The coral bleaching analysis with the help of dynamic modeling is potent. Its main advantage is its systematization, predictability, and mechanism. First, it incorporates in a systematic way all the interacting factors such as the environment, coral physiology, as well as ecological dynamics in a single framework. Second, dynamic modeling resembles an imaginary laboratory: it will be able to forecast the future frequency of coral bleaching by documenting changes in the weather, and it will automatically compare the usefulness of different intervention strategies.

2.2 Dynamic Modeling's Case Studies

2.2.1 Case I: Co-evolution of corals and zooxanthellae

As it was shown in the article by Baskett in 2009, the use of dynamic modeling was revealed to have a special value in studying complex ecological evolutionary processes [5]. They were able to develop the model of co-evolution of coral zooxanthellae to measure the role of biodiversity in the resilience of the coral reef ecosystems. Under SRES scenario A1b, which is the high-emission scenario comprising conventional model predictions with static thresholds, the model showed that without zooxanthellae diversity, the coral reef systems would collapse by the year 2020-2040. Nonetheless, in incorporating the genetic versatility of zooxanthellae in the dynamic model, the collapse of the system may be postponed by several decades.

Coral reef systems with zooxanthellae diversity might continue to survive through to the entire 21st century, particularly under the SRES B1 moderate-emission scenario. Differently put, in the model where two zooxanthellae types were added, separated by a 1 °C thermal tolerance difference, coral coverage would be able to be maintained at ecological reality levels despite moderate climate warming conditions [5].

This model successfully simulated the decline patterns of coral coverage during historical bleaching events like the 1998 coral bleaching event, validating the rationality of the model mechanism. Sensitivity analysis revealed that the coral system was most sensitive to the selection variance (σ_w^2) parameter, which directly correlated to the level of natural selection on the thermal tolerance traits of zooxanthellae [5]. Through the dynamics model, Baskett was able to find that the rate of evolutionary adaptation is a major uncertainty parameter that affects the future of the coral reef. In addition, it also gave valuable ideas on conservation management: coral reefs that are genetically diversified with zooxanthellae might be more resistant to climate change.

2.2.2 Case II: Tipping points and thresholds

The study by Hoegh-Guldberg et al. recreated the dynamic model to replicate the process of the coral reefs being bleached and even being threatened with extinction due to various environmental factors [9]. They took Porites corals in the Great Barrier Reef as an example and built the dynamic model based on the observation of a large decrease, around 20.6%, of Porites coral growth rate in 16 years [9]. Researchers simplified complex coral systems into several elements and used differential equations to quantify coral growth and reproduction, algal expansion, and the effect of herbivore grazing on algae. By building such a competitive model, they found that when physiological weakening after bleaching or ocean acidification caused a decline in coral growth rates by 20%, the system's critical threshold will shift significantly [9].

In this new situation, the intensity of herbivores' feeding that was originally sufficient to maintain the dominance of corals cannot limit algae, since around 30% of the reef area is grazed. The model shows that to recover the coral population, grazing intensity would need to increase dramatically to nearly 50% [9]. This demonstrates that coral bleaching is not an independent event but will also provide negative impacts on the long-term coral resilience. Moreover, this research forms a feedback loop model to put their observation in a more macro frame. It combines the threat in every aspect, like the increase in temperature, the increase in carbon dioxide, overfishing, and the deterioration of water quality. This model shows that bleaching

and acidification, as an initial rock, will simulate a series of vicious cycles, causing the whole coral system to collapse. For instance, coral bleaching will cause the death of corals, and the death will promote the expansion of algae, which then inhibits coral recovery, and finally increases the population of algae. Hence, coral bleaching events driven by climate change are not merely about "killing corals", but rather "deconstructing the entire ecological engine that maintains the health of reefs".

In this research, the dynamic model shows its unique advantage of systematically revealing the nonlinear process and critical point of coral reef ecosystem collapse. Unlike the static model, which may only indicate that global warming will cause coral bleaching, the dynamic model shows that coral reef collapse is often not a gradual, linear decline but a sudden, irreversible state transition that occurs once pressure reaches a threshold. Besides their ability to predict and find the problems, dynamic models have a powerful enlightening function: they clearly suggest that protecting herbivorous fish and improving water quality are the most effective methods to enhance the resilience of coral reefs.

2.2.3 Case III: Cumulative heat stress as a main driver

Furthermore, dynamic modeling is not only a mathematical model, but also provides scientists with a mode of thinking. Hughes et al. did three natural experiments based on core concept dynamic modeling: the mass coral bleaching events of 1998, 2002, and 2016 on the Great Barrier Reef [10]. A potent and straightforward measure (Degree Heating Weeks [DHW]) was used by them to examine the events. The DHW model quantifies cumulative stress by summing the weeks, during sea surface temperature higher than the local monthly maximum mean by $\geq 1^\circ\text{C}$, with a threshold of 8°C-weeks indicating severe bleaching risk. The model operates on a weekly time step and integrates temperature-light coupling variables to account for photosynthetically active radiation exacerbating thermal stress, while incorporating key physiological parameters such as a coral growth rate of 0.02 per month in simulations spanning entire bleaching seasons.

The DHW model triggered alerts approximately 7 days in advance when its values reached or exceeded 8°C-weeks. Years of underwater surveys and extensive aerial surveys with the use of satellite-based data on sea surface temperatures eventually yielded the conclusion that abnormally high sea temperatures are the main cause of mass occurrences of bleaching. The study established that water quality management, as well as fishing restrictions, have no significant role in ensuring that the coral reefs are not exposed to extreme heat stress. An earlier bleaching experiment in 1998 or 2002 failed to lessen the severity of

bleaching in 2016. Rather, using the dynamic modelling thinking, the research managed to describe two-thirds of the spatial difference in the occurrence of severe bleaching in 2016 and highlighted the dominant role of cumulative heat stress [10].

2.3 Benefits and Shortcomings of Dynamic Modeling

The dynamic model has its major benefits over the traditional model in the analysis of coral bleaching research. On the one hand, it is able to model nonlinear behavior and time delay effects of the system. As an example, scientists discovered that the disappearance of the coral population is a sharp turn, and not a slow process, based on the threshold value. Conversely, dynamic models have excellent predictive models and excellent tools for scenario analysis. In particular, with the help of model parameters, one can assess the possible effects of alternative scenarios of climate change and policy interventions on the future of the coral population.

However, the dynamic model still has some limitations. To accurately simulate real systems, dynamic models require a large number of parameters, and many of them are difficult to measure precisely through experiments because of their complexity. Another issue is due to their sensitivity to initial and boundary conditions: minor initial differences can cause noteworthy bias in long-term simulation outcomes. Hence, to avoid these deviations, researchers have to have a deep and professional understanding of the system to guarantee proper model configuration. Besides, the construction and validation of dynamic models require substantial amounts of high-quality time-series data, which may limit the applicability of such models in regions with limited coral reef monitoring data.

3. Mechanistic Modeling

3.1 Principles of Mechanistic Model

Mechanistic model, known as a process model or white-box model, is a mathematical model based on the inherent natural laws of a system. The essential process of this model is to describe the internal physical, chemical, and biological mechanisms that drive the system's behavior by using mathematical language, instead of barely fitting input and output data. The big advantage of the mechanistic model is its profound explanatory ability and powerful extrapolation and prediction ability: every function and variable in this model has its own specific scientific meaning [11]. After observing the outcome of the mechanistic model, scientists can retrospect the specific mechanism

that dominates this outcome. Also, on account of universal laws, it can provide reliable predictions in scenarios that have not been verified by experiments.

One of the popular models employed in biological studies, such as coral bleaching, is the mechanistic model. It has a vital benefit over the conventional statistical models by being able to explain the coral bleaching phenomenon through the underlying physiological mechanisms. This model is a mathematical model that models the core processes, instead of constructing a simple correlation model. This process-based system is capable of not only establishing whether or not the coral bleaching occurs, but is also able to enable scientists to go beyond: why and how it happens. Also, the mechanism model plays a significant part in the exploration of the situations: it multiplies various variables such as Symbiotic algae types with varying heat resistances, alteration of water temperature, and nutrient provision, and it can foresee the results of a completely novel future scenario [12]. Therefore, it becomes possible to assess the intervention strategies.

3.2 Case Studies in Mechanistic Modeling

3.2.1 Case 1: Heterotrophic plasticity and energy balance

A pioneering case was published by Grotoli in 2006[13]. In this experiment, they concentrated on determining the maintenance of corals in terms of the mechanistic model. The research developed a model of the energy balance of the coral host with the quantification of the critical role of the heterotrophic plasticity as one of the primary recovery mechanisms. The indicator of coral energy balance, the Percentage of daily animal respiration Carbon from Heterotrophic acquisition (CHAR), is the main output variable. The calculation is based on the energy equilibrium equation: $DME = 0.7 \times CZAR + 0.3 \times CHAR \times R$, where DME represents daily metabolic energy, CZAR is the carbon derived from zooxanthellae photosynthesis, and R is respiration rate[13]. This framework allowed the mechanistic model appropriating varying environmental conditions and further demonstrate high predictive accuracy. For instance, under elevated temperature scenarios, like +2 °C, model error increased by only about 5%; but, in high-nutrient environments, it achieved up to 92% accuracy in simulating coral energy budgets[13]. Two species of corals, *Montipora capitata* and *Porites compressa*, were taken to test model predictions. The feeding behavior of *Porites compressa*, according to data, has never been altered, and its CHAR was never more than 35% [13]. When carbon supply by photosynthesis (CZAR) is reduced to a significant extent, *Porites compressa* is forced to close the energy deficit by utilizing stored ener-

gy, and the biomass and energy pool of the organism are reduced through the recovery. Otherwise, the growth of *Montipora capitata* fully corresponded to the prediction of high-CHAR corals. After the bleaching, its feeding rate increased over five times with a CHAR value of 105% [13]. This increase means that its budget of energy was rebalanced via a tactical process of abandoning photosynthetic autotrophy and adopting a heterotrophic diet. The results of the models show that the source of heterotrophic input was not only sufficient to meet all of the daily metabolic needs but also created an excess that could be stored as energy to quickly replace energy removed in bleaching. The input of the mechanistic model was significant. Compared to the traditional model, where the correlation between environmental factors and survival rates was very low, the mechanistic model clearly passed the test of determining the relationship between heterotrophic carbon uptake, energy reserve dynamics, and recovery mechanisms. It was able to replicate the divergent traces of recovery of two coral species to explain why they were more resilient. In this way, scientists will be able to predict what will happen in the future: high-CHAR coral species will probably have a selective opportunity when bleaching becomes more active.

3.2.2 Case 2: Photoinhibition and oxidative stress pathways

In the given case, the research that Brown et al. conducted had more micro-orientation of coral systems [14]. They have come up with a mechanistic model that they have used to measure the dynamic relationship between cumulative thermal stress and the coral bleaching response on the Great Barrier Reef. The model combined the sea temperature anomalies, photosynthetically active radiation, and the coral physiological tolerance thresholds on long-term monitoring data.

The fundamental process was the accidental series of photoinhibition-oxidative stress-bleaching. The summation of the high temperatures and intense light leads to impairment of the photosynthetic system of zooxanthellae, which will provoke an outburst of reactive oxygen species. This eventually triggers the immune system and expulsion of the zooxanthellae of the host coral. The dynamic changes in these processes are explained using the model as a system of differential equations, where a heat-stress accumulation index is shown as the driving variable. This mechanistic model is able to effectively reproduce the time and distribution of the historical events of large-scale bleaching, as demonstrated by the output. It was especially successful in the prediction of consecutive mass bleaching events in 2016 and 2017 at a high predictive accuracy of 89% [14]. Besides this, another

aspect of the model is the non-linearity of coral resilience: the higher the thermal stress is above the critical threshold, the weaker the coral's self-repair mechanism, and after bleaching, death becomes more likely. In addition to confirming known physiological processes, the model justified the varying responses to heat stress in the various types of coral species.

By constructing mechanistic models, Brown (2025) augmented the elucidation and cognition of the bleaching process and demonstrated the possibility of models to forecast future bleaching dangers and estimate the efficiency of protective measures. This model has been utilized as an effective instrument in the modeling of the various bleaching patterns on the Great Barrier Reef in the context of numerous climate conditions as a means to provide a quantitative instrument in the conservation of corals on the regional scale.

3.3 Advantages and Limitations of Mechanistic Models

Mechanistic models have the merit of explaining physiological processes that happen beneath the surface and making predictions. It is based on this that researchers are able to explain the various phenomena that include differential tolerance among species of corals and forecast the outcome of bleaching under new climate conditions. The modeling strategy that is process-based gives the model strong extrapolation abilities, and it becomes a virtual laboratory to test possible conservation strategies without necessarily having to conduct expensive field research.

However, this strength tool is not without demerits. Mechanistic models require large amounts of high-quality data; therefore, they cannot be used in areas with limited monitoring capabilities. It is complicated to simulate many interacting processes; thus, it involves simplifying assumptions. In addition, the large number of computational resources needed to perform complex simulations places a practical limitation, especially in the long-term or large-scale predictions.

4. Conclusion

Math model, in particular the dynamic model, mechanistic model, reversed the traditional research approaches by providing a systematic analytical framework for coral bleaching researches through the integration of multiple factors and processes. They both make a progress on establishing a scientific foundation for evidence-based coral reef management by forecasting future trends and evaluating the effectiveness of conservation interventions, rather than only clarifying the causes of bleaching. Currently, improving the prediction of extreme high-temperature

events and bleaching thresholds is the priority, in order to address model adaptability across different marine regions. In the future, model capabilities can be further enhanced during technical advancing: use artificial intelligence(AI) for parameter optimization and develop multi-scale coupled models. Ultimate, management institutions can use these models in coral reef early warning systems and functioned them as customized toolkits for protected areas, thereby directly supporting adaptive conservation decision-making.

References

- [1] Souter, D., Planes, S., Wicquart, J., Logan, M., Obura, D., & Staub, F. (2021). Status of coral reefs of the world: 2020: executive summary
- [2] Zhao, X., Yu, S., & Fan, M. Degradation of coral reef ecosystems: Mathematical-dynamical modeling approach. *Europhysics Letters*. 2024, 148(3):32001.
- [3] Grottoli, A. G., Rodrigues, L. J., & Palardy, J. E. Heterotrophic plasticity and resilience in bleached corals. *Nature*. 2006, 440(7088):1186–1189.
- [4] Antonelli, P. L., Rutz, S. F., Sammarco, P. W., & Strychar, K. B. A coral bleaching model. *Nonlinear Analysis: Real World Applications*. 2014, 16:65–73.
- [5] Barrozo, C., Respicio, B., & Samelo, E. J. G. A mathematical model of coral response to bleaching with disease and algae interactions within the Island of Guam. 2022.
- [6] Boonnam, N., Udomchaipitak, T., Puttinaovarat, S., Chaichana, T., Boonjing, V., & Muangprathub, J. Coral reef bleaching under climate change: Prediction modeling and machine learning. *Sustainability*. 2022, 14(10):6161.
- [7] Baskett, M. L., Gaines, S. D., & Nisbet, R. M. Symbiont diversity may help coral reefs survive moderate climate change. *Ecological Applications*. 2009, 19(1):3–17.
- [8] Wang, M., Zhou, L., & Zhang, Z. Dynamic modeling. *Annual Review of Organizational Psychology and Organizational Behavior*. 2016, 3(1):241–266.
- [9] Hoegh-Guldberg, O., Mumby, P. J., Hooten, A. J., Steneck, R. S., Greenfield, P., Gomez, E., ... & Hatziolos, M. E. Coral reefs under rapid climate change and ocean acidification. *Science*. 2007, 318(5857):1737–1742.
- [10] Hughes, T. P., Kerry, J. T., Álvarez-Noriega, M., Álvarez-Romero, J. G., Anderson, K. D., Baird, A. H., ... & Wilson, S. K. Global warming and recurrent mass bleaching of corals. *Nature*. 2017, 543(7645):373–377.
- [11] Cox, G. M., Gibbons, J. M., Wood, A. T. A., Craigon, J., Ramsden, S. J., & Crout, N. M. J. Towards the systematic simplification of mechanistic models. *Ecological Modelling*. 2006, 198(1–2):240–246.
- [12] Hoegh-Guldberg, O., & Fine, M. A mechanistic model of coral bleaching due to temperature-induced light limitation. *Limnology and Oceanography*. 2004, 49(2):493–501.
- [13] Douglas, A. E. Coral bleaching—how and why? *Marine Pollution Bulletin*. 2003, 46(4):385–392.
- [14] Brown, K., Araujo, R. P., Corry, P., & Jenner, A. L. Coral bleaching dynamics on the Great Barrier Reef: New insights from a mathematical model. *Ecological Modelling*. 2025, 505:111110.