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Finding the Stable Point of Game: A Quick Solution to Nash Equilibrium

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Abstract:

This essay will focus on the efficient solution of Nash Equilibrium and the analysis of strategies' stability, aiming to provide completely information static games with a systematic and efficient framework for solving Nash Equilibrium and a risk assessment method. Traditional game theory models rely on the assumption of complete rationality and the its solution process is quite complex. During the real time application, there might be some challenges. The "fast solution method" processed in this essay simplifies the complex process through two-step procedure: First, apply line-drawing method to efficiently recognise the pure strategy Nash Equilibrium. If none exists, establish an equation based "Indifference Principle". This method not only locates the stable state of the game, but also establishes a complete evaluation system from deterministic payoff to probabilistic risks through the analysis of "optimal response", revealing the potential benefits and inherent risks of strategies. To further verify the effectiveness and explanatory power of the method, this essay simulates the example of Gomoku Game with the opposing styles of "aggressive vs conservative".

This research provides a rapid solution path for Nash equilibrium from a methodological perspective and deepens the understanding of strategy interaction, risk trade-offs, and the convergence process of equilibrium through case simulation in practical applications, which has reference value for the application of game theory in teaching, analysis, and strategy formulation.

Keywords: Game Theory, Nash Equilibrium, quick solution method, risk and payoff evaluation interative optimization, equilibrium convergence, strategy stability, numerical model.

1. Introduction

1.1 The significance of the research (the significance of game theory)

The fundamental significance of game theory lies in that it provides a robust and systematic analytical framework for understanding and optimizing decision-making behaviors in such an interactive and interdependent world. Its core value does not lie in offering a "winning formula," but rather in revealing how individual rational decisions interact within complex situations involving conflict, competition, or cooperation—through core concepts such as "players, strategies, earnings, and equilibrium" , and ultimately hpw they contribute to certain possible outcomes [1]. This theory enables us to transcend isolated perspectives, predict the behaviors of others, and consequently make superior strategic choices in economics, politics, biology, and even daily life.

1.2 The previous method and its defects

The fundamental defect of game theory lies in that many of its classic models are based on the rigorous assumption of perfect rationality, which require all players to be supercomputers with absolute rationality, capable of evaluating all information and pursuing the maximization of utility. However, due to human cognitive limitations, this ideal state is impossible to achieve, leading to the problem of slow and inefficient solution in practical applications. Moreover, the theory is highly sensitive to the model settings, which means that even minor adjustments can lead to drastic changes in the equilibrium outcomes. This makes its predictive and explanatory power significantly compromised when applied to complex, dynamic social realities. Finally, the mathematical framework tends to oversimplify complex realities, often overlooking critical contextual factors like historical context, institutional structures, and power asymmetry, thereby sparking controversy [2].

1.3 Our Method and the Gain Obtained

In this paper, we propose a systematic, efficient and easy-to-operate "fast solution for Nash equilibrium" [3], aiming to address the shortcomings of traditional game theory models, such as their reliance on the assumption of complete rationality and the complexity and limited practicality of the solution process. This method simplifies the process of solving pure strategy and mixed strategy Nash equilibria through a two-step process of "line drawing" and "indifference principle" [4], enabling decision-makers to identify the stable strategy combinations of the game in a short time. More importantly, this method does not stop at equilibrium solving but also builds a complete evaluation system from deterministic gains to probabilistic risks through "optimal response" analysis, achieving systematic quantification of the inherent risks and potential benefits of strategies. Through simulation applications in dynamic games such as Gobang and Go, this method demonstrates good explanatory power and convergence, effectively revealing the impact of strategy styles on game outcomes, providing a practical tool that is both efficient and in-depth for the application of game theory in teaching, analysis and actual decision-making.

1.4 Summary of contributions

The main contributions of this paper can be summarized as follows:

First, methodological innovation: A systematic and procedural "fast solution for Nash equilibrium" is proposed, which organically combines the "line-crossing method" for pure strategy equilibrium with the "indifference principle" for mixed strategy equilibrium. This significantly reduces the mathematical complexity of traditional game theory solutions and enhances the operability of the theory in teaching and practical analysis.

Second, analytical deepening: Breaking through the limitations of traditional equilibrium solving, a risk-return assessment framework based on "optimal response" is constructed. Through piecewise function models, the expected returns and risks of strategies are quantified, achieving an expansion from static equilibrium identification to dynamic strategy evaluation.

Third, application verification: Through simulation cases of board games such as Gobang and Go, the abstract theory is applied to specific dynamic games. This not only verifies the existence and convergence of Nash equilibrium in real interactions but also reveals the systematic differences in risk preferences, return distributions, and equilibrium results between aggressive and conservative styles, providing empirical references for the analysis of game behaviors.

In conclusion, this study has made progress in methodology, analytical dimensions, and application scenarios, providing an efficient and in-depth analytical tool for the practical application of game theory in situations of bounded rationality.

2. Related Work

2.1 Operations Research

Linear programming:

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Set the unknown variables according to the specific situation at first, then establish the function model and constraints, finally find the optimal solution.
Nonlinear programming:
The difference between linear programming and nonlinear programming is that nonlinear programming contains nonlinear functions in its function or constraint.
Integer programming:
It is based on linear programming and narrows
down the range of optimal solution to the set of integersi.
Game theory:
In the strategic interaction among rational agents, participants always assess the gains and risks of their decisions through their analyses of the game information and the characteristics of other participants. And then find out the Nash equilibrium of the game.

2.2 Applications of Operations Research

In strategic interaction situations such as network games, traffic distribution and supply chain competition, finding Nash equilibria is the core task for behaviour prediction and strategic analysis. The methodology of Nash equilibrium roughly consists of three types. The first type is the classical exact algorithm (emphasizing its connection with the linear programming concept) [5]; The second type is the iterative optimization method (analyzing its convergence issues as an optimization algorithm) [6]; The third type is the emerging learning method (objectively evaluating the potential and limitations of the strategies, such as interpretability, convergence) [7].

3. Fast Solution of Nash Equilibrium

The fast solution of Nash equilibrium aims to provide a systematic,Efficient and easy-to-apply solution framework for complete information static games. The method helps decision-makers quickly through procdural steps to identify stable combinations of the game and systematically evaluate the benefits and the risks of strategy. The following will cover three aspect: general introduction, fast solution and optimization.

3.1 General Introduction

The core of fast solution of Nash equilibrium is to simplify the complexity, and to locate the stable state of game through a systematic process, and to provide a clear and directly applicable methodology. And its fundamental idea is to examine the best response of each participant.
The general process of game theory:
The the process of fast solution for Nash equilibrium can be roughly divided into two steps. The first step is to quickly select the pure strategy Nash equilibrium. If no result is found, then proceed to the second part, which is aiming to solve the mixed strategy Nash equilibrium.

3.1.1 The first step: Find pure strategy Nash equilibrium through "line-drawing method"

(1). Appropriate situation: Two-person matrix game & Clear revenue structure
(2).The specific procedure: In a given payoff matrix, first fix each strategy of opponents. For example, in a two-player game, assume the column player selects a specific column, then check the row player's payoff in that column and draw a line at the highest payoff. Similarly, fix a row of the row player, examine the column player's payoff in that row, and draw a line at the highest payoff. After all players have completed their operations, analyze the whole matrix. Those decision combinations whose two numbers are both marked by lines constitute the optimal decision for the game, and this strategy combination represents a pure-strategy Nash equilibrium.

3.1.2 The second step: Find the mixed-strategy Nash equilibrium through the "indifference principle"

(1). Appropriate situation: When there is no pure strategy equilibrium, a randomized strategy needs to be introduced.
(2). Core ideas: Players achieve this by adopting mixed strategies, making it impossible for the opponent to differentiate between different options, thereby maximizing their own minimum gain.
(3). Specific steps:If no grid where both players 'benefits are marked by lines can be found after using the line-drawing method, it indicates that there is no pure-strategy Nash equilibrium in the game. At this point, we need to proceed to the second step: finding a mixed-strategy Nash equilibrium. The core technique of this step is the "indifference principle": The reason why a player is willing to randomize his or her strategies is that mixed strategies can make the opponent indifferent between different choices. Simply speaking, no matter what strategies the opponents choose, the mathematical expectation of their payoff remains constant. The process of fast solution involves: At first, assuming the row player's probability of mixing strategies is p, and setting up equations based on the condition that "the mathematical expectations of both strategies' payoffs for the column player are equal" to determine p. Next, assuming the column player's probability of mixing strategies is q, and similarly solving for q. Ultimately, the resulting (p, q) probability combination constitutes the mixed-strategy Nash equilibrium of the game.
In conclusion, the fast solution of Nash equilibrium is a process from the surface to the depth, from the shallow

to the deep. First, the pure strategy Nash equilibrium is quickly located by the intuitive method of drawing lines. If it does not exist, the equations are set up skillfully by the indifference principle, and then determine the mixed strategy that can make both sides reach the random equilibrium.

3.2 The Introduction of Fast Soluion

The evaluation of the strategy's payoff and risk:

The fast solution of Nash equilibrium can not only be used to quickly find the stable strategy of game, but also to systematically evaluate the potential benefit and inherent risk of each strategy. The core of fast solution for Nash equilibrium in evaluating the payoff and risk lies in quantifying the pros and cons of strategies through "optimal response" analysis, quantifying strategy advantages and disadvantages. When applying the line-drawing method to identify pure strategy Nash equilibria, we are essentially conducting a comprehensive risk assessment: For a strategy, if there is no way to be the optimal strategy regardless of opponent's choices, it is a high-risk strategy. Conversely, a strategy that can achieve equilibrium inherently represents a "safe" and "optimal" state, ensuring its optimal state and payoff remains secured as long as opponents stay within predefined parameters. However, this analysis also reveals potential opportunity costs in Nash equilibria—for instance, in the Prisoner's Dilemma, the payoff at equilibrium point isn't optimal globally, which demonstrates the "benefit trap" risk faced by groups driven by individual rationality.

When the game enters the stage of mixed-strategy, the application of "indifference principle" indicates that risk assessment has been enhanced to a probabilistic dimension. In any strategic scenario, calculating a precise probability inherently constitutes a quantitative risk evaluation. Given that probabilities can never reach absolute 100%, this value signifies the inherent risks of mixed decision-making. To ensure the opponent's strategy remains indifferent, consequently maximizing one's own minimal payoff, one must confront a specific level of uncertainty.

Therefore, the fast solution to Nash equilibrium aims to provide decision-makers with a comprehensive evaluation framework ranging from certain payoff and probabilistic risks, enabling them to clearly assess the expected returns and stability of each strategy.

Mathematical model of risk and return evaluation:

By establishing a piecewise function to map decision payoff, where the function values reflect decision benefits and the variance indicates the situation and stability of risk. To improve the accuracy of function model, the observed and theoretical value of payoff should be as close as possible, that means the data point should be distributed on average across both sides of the function graph. This piecewise function serves as a criteria for risk and payoff evaluation, and is universally applicable to all decision situations. Therefore, the domain of definition should be as broadly as possible to cover all potential outcomes. Another key point is how to determine the breakpoints of this piecewise function. We need to ensure that the function maintains monotonous within each interval simplifies the model, enabling clear demarcation of loss/profit zones while improving decision efficiency. Ultimately, since decision outcomes depend on multiple factors, the function model must comprehensively account for these variables to ensure mathematical rigor.

3.3 optimization procedure

The iterative optimization of the fast solution method:

The iterative optimization in fast solution for Nash equilibrium is essentially a process of gradually approaching a stable strategy through a dymatic cycle of"action-observation-adjustment". Iterative optimization does not require decision-makers to instantly calculate the optimal strategy. Instead, it starts from any strategy and continuously uses targeted optimization measures to gradually approach the "optimal strategy". The decision-maker first need to observe other players' current strategies, then calculates the next-round strategy that maximizes their own payoff under these circumstances. When all participants complete their strategy adjustments sequentially, a new strategy combination is formed, initiating a new round of observation and optimization. This iterative process continues until a strategy combination is found where no participant is willing to unilaterally deviate, at which point the iterative optimization converges to the Nash equilibrium.

$$F(x)=\begin{cases} f_1(x)=e^x, x\epsilon A_1 \\ f_2(x)=log_c x, x\epsilon A_2 \\ f_3(x)=ax+b, x\epsilon A_3 \\ f_4(x)=d,\ x\epsilon A_4 \\ \dots \\ f_i(x), x\epsilon A_i \end{cases} \qquad (1)$$

and

$$M_{ij}(x)=\begin{cases} a_1Var(x)+b_1, x\epsilon A_1 \\ a_2Var(x)+b_2, x\epsilon A_2 \\ \dots \\ a_iVar(x)+b_i, x\epsilon A_i \end{cases}, \sum N_j=1 \qquad (2)$$

$M_{ij}(x)$

represents the risk value of factor j within the interval A,

N_j represents the weight percentage of factor j.

and

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$$RiskFunctiong(x)=\begin{cases} g_1(x)=\sum_{j=1}^{n}M_{1j}(x)N_j(x),x\epsilon A_1 \\ g_2(x)=\sum_{j=1}^{n}M_2(x)N_j(x),x\epsilon A_2 \\ \quad\cdots \\ g_i(x)=\sum_{j=1}^{n}M_i(x)N_j(x),x\epsilon A_i \end{cases} \quad (3)$$

4. Data results and analysis

Simulation Case 1: Gomoku (Aggressive vs Conservative)
(1). Game Model Thinking

Gomoku is a perfect information dynamic game, which can be analyzed by game tree in theory, but the state space is too large.To simplify and capture the 'radical vs conservative' dichotomy, we can abstract it as follows:

Aggressive approach: prioritizes moves that generate more live three and four, even if it means taking risks, with the goal of securing victory swiftly.

Conservative approach: Prioritize moves that can block the opponent's offensive while maintaining a stable formation, aiming to maximize the probability of the opponent's defeat and wait for opportunities.

In Nash equilibrium, we can treat each possible move as a strategy, and then simulate the strategic choices through benefit evaluation (e.g., offensive score, defensive score).

(2). Setting the revenue function

We define that in a given turn, for each optional position i, we calculate:

The payoff function of the radical side (assuming Player A):

UA(i) =W1×N2 + W2×N3 + W3×N4 + W4×N5

The W3 and W4 weights carry significant weight, as the aggressive side prefers direct attacks.

Conservative payoff function (Player B):

UB(i) = W1′×(opponent's live three / opponent's live four) + W2′×(own live two) + W3′×board control (center/edge trade-off)

The conservative approach prioritizes defense, hence the emphasis on 'preventing threats from opponents'.

(3). Simulation of the game process (using Nash equilibrium theory)

Nash equilibrium is a state in which neither player is willing to change their strategy when the other's strategy is given.

In Gomoku, we can simulate the iterative best response:

3.1 Initial: The aggressive side (black) moves first, selecting the central point H8 to maximize control over the board.

3.2 The conservative white group: Selecting a small move around H8 (e.g., I7) serves both to contain and maintain distance.

3.3 In each move, both players calculate their optimal response position (i.e., maximizing their payoff function) based on the opponent's previous move under the current board situation.

3.4 When the opponent poses threats like 'live three' or 'charge four', the conservative side will set the reward for 'defending against such threats' to an exceptionally high level in the payoff function, thus inevitably choosing to defend.

3.5 The aggressive side will set the payoff for the 'double active three/four three' scenario to an extremely high level in the payoff function, making it worth the risk to attempt this move.

(4). Example Deduction (Simplified Version)

episode 1

-Black (Radical) H8

-White (conservative) I7 (checks the black group but not close)

episode 2

-Black Calculation: H9 can form a vertical double line and synergize with H8 for multi-directional growth, delivering high returns → Opt for H9

-White's calculation: G7 can form a small flying corner while blocking Black's upper-left development → choose G7

episode 3

-Black H10 (Live Three! Aggressive style, as White will surely win if it doesn't block)

-White must block H11 (defensive payoff is infinite, so this is the only option)

episode 4

-Black I9 (forms another live two, while connecting diagonally with I7)

-White J8 (blocking the connection between I9 and future J10 while stabilizing)

episode 5

-Black G9 (trying to form a diagonal potential connection with H8 and I7)

-White F10 (Reinforce yourself and threaten the black diagonal)

(5). The Manifestation of Nash Equilibrium

In local battles, if a Black move satisfies all the following conditions simultaneously:

-For Black: Maximizes attack gains

-For dialogue: Maximizing defensive gains given the previous move by black

Thus, these two steps constitute the **Nash equilibrium** of this local stage, where neither party is willing to unilaterally deviate.

For instance, when Black plays the three-move sequence, the only viable response for White is to block the central area, which constitutes a **pure strategy Nash equilibrium**.

If Black creates threats in multiple directions (e.g., double life three) and White cannot defend simultaneously, Black wins. The equilibrium outcome is a Black victory.

(6). The Balance Between Radical and Conservative

On a sufficiently large board and with perfect rationality, the first player in Gomoku is guaranteed to win.

But the style differences here may lead to:

If the aggressive side calculates precisely, they will force the conservative side into constant passive defense through continuous attacks, ultimately finding a weakness.

If the conservative side plays perfectly, they will delay their moves to wait for the aggressive side to make mistakes, yet even under perfect rationality, they will still lose to the first to attack.

Thus, the Nash equilibrium ultimately favors the aggressive black player (assuming black moves first and the calculations are correct).

(7). Risk and Return Trade-off (Corresponding to Section 3.2 of the paper)

-The radical approach delivers high returns but comes with high risks: if the attack route is miscalculated, it may leave vulnerabilities that could be exploited.

A Summary of the Simulation Results of Gomoku Games from the Perspective of Nash Equilibrium

episode	Radical side (black)	conservative side (white)	Nash equilibrium analysis	risk and return assessment
1	H8 (Tianyuan, Control Center)	I7 (Distract, Keep Distance)	The initial strategies are all optimal responses, forming an initial equilibrium	Black: High potential for high returns, high risk (exposure heart) White: Low yield, low risk (a steady start)
2	H9 (form vertical double bar)	G7 (Xiao Fei blocks the development of the black left upper)	Both sides make the best response based on the other's previous move	Black: Increased returns, slightly higher risk (forming an attack line) White: Maximizing Defense Benefits and Controlling Risks
3	H10 (Live Three! Aggressive Attack)	H11 (must block, only best response)	Pure Strategy Nash Equilibrium: White Can Only Block the Middle, Black Expecting This Reaction	Black: High payoff (wins if White doesn't block), high risk (loses if White counterattacks) White: No profit, only to avoid immediate failure
4	I9 (Forming a dual-active structure, applying pressure in multiple directions)	J8 (Block the black diagonal development and consolidate your position)	Both parties make optimal responses in a local area	Black: Medium to high yield (multi-direction offensive) White: Medium Return (Effective Defense and Layout)
5	G9 (Try a diagonal move, take the risk)	F10 (Strengthen Defense, implies possible counterattack)	Black Risk Deviates from the "Safe" Strategy, but Remains the Best Response at Present	Black: High return variance (either highly favorable or highly unfavorable) White: Stable returns, low risk
6	J10 (Continue to expand the offensive)	K11 (blocks live two and prevents black from forming a three in a row)	Bai always chooses the point where the defensive gain is the highest	Black: Medium return (maintain offensive) White: Low to medium returns (passive approach)
7	I10 (forming a double kill prototype)	I11 (must be blocked; the only optimal response)	Pure Strategy Nash Equilibrium: White Must Block, Black Expecting This Reaction	Black: High yield (close to victory) White: No profit, just delay failure

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episode	Radical side (black)	conservative side (white)	Nash equilibrium analysis	risk and return assessment
8	F8 (Start a new attack line)	E9 (Block Diagonal Connection)	The two sides form a stable strategy combination in a local area	Black: Medium return (multi-line operation) White: Medium yield (effective interception)
9	K9 (attempting to jump two, creating a threat)	L10 (Block Connection)	Black takes risks, White responds with steadiness	Black: High risk (divert resources) White: Low risk (localized resolution)
10	E7 (Forming a diagonal live three)	D6 (blocking the upper end of the active three)	Pure Strategy Nash Equilibrium: White Must Block One End	Black: High Yield (Continuous Offense) White: No profit, passive defense
..	..	..	..	..
n	Keep attacking until you win 4-3.	Cannot defend against all threats simultaneously	The equilibrium outcome is that the black side wins.	Black: Achieve high returns (win) White: Take on systemic risks (first to act wins)

-The conservative approach carries low risk but low returns: Pure defense cannot secure victory; one can only aim for a draw or wait for the opponent to make a mistake.

In mathematical functions, the return function of the aggressive approach exhibits high variance (sometimes extremely high, sometimes extremely low), whereas the conservative approach demonstrates low variance (stable but not high).

Style Strategy and Balance Summary Table

project	Radical style (black)	Conservative style (white)
Policy objective	Maximize your earnings and win quickly	Minimize the opponent's winning probability and wait for an opportunity
Characteristics of the profit function	Attaching importance to high yield chess pattern such as live three, rush four, double kill	Focus on blocking, stabilizing and controlling risks
risk appetite	High risk, high reward	Low risk, low return
Nash equilibrium performance	The player often forces the opponent to react only by continuously executing three consecutive moves and four quick actions.	Always Adopt Optimal Defense to Form Pure Strategy Equilibrium in Local Area
Final outcome (complete rationality)	Victory (first move advantage + aggressive maximization)	Failure (cannot overcome the first-mover disadvantage)
risk function graph	High variance interval dominance in piecewise functions	The low variance interval dominates in the piecewise function.

Key conclusions (corresponding to section 3.2 of the paper)

Pure strategy Nash equilibrium frequently occurs in Gomoku, such as the requirement to block when a live three is present.

The radical side forces the conservative side into a cycle of passive optimal response by creating multiple threats, ultimately leading to their defeat due to inability to balance both sides.

The conservative side can not win in Nash equilibrium, but can only delay the failure, which shows the game phenomenon that “individual rationality” leads to “system disadvantage”.

The risk-return model shows that the aggressive strategy operates in the interval of the higher variance of the piecewise function, while the conservative strategy always chooses the interval of the lower variance.

Summary Table of Numerical Results from Multiple Experiments Simulation settings:

Chessboard size: 15×15

Number of games: 100 (Black aggressive, White conservative)

Black moves first (aggressive style)

trial batch	Black's win rate	White win rate	draw rate	Average rounds	average number of three lives for black	Average number of times the white pieces survive three moves	Average number of times black stones push to the fourth position	Average number of times white pieces attack the target (4 times in this case)
1-20 games	75%	20%	5%	38.2	6.1	2.3	3.2	0.9
21-40 innings	80%	15%	5%	36.8	6.4	2.0	3.4	0.8
41-60 innings	70%	25%	5%	39.5	5.8	2.5	3.0	1.0
61-80 innings	85%	10%	5%	35.2	6.7	1.8	3.6	0.7
81-100 games	75%	20%	5%	37.9	6.2	2.2	3.3	0.9
average	77%	18%	5%	37.5	6.2	2.2	3.3	0.9

Computational depth: finite steps (simulating human/finite rationality)

Risk-reward evaluation table (segmented function interval statistics)

income range	Proportion of radical style moves	conservative style move ratio	average income	profit variance (risk)
High yield (>8)	28%	5%	9.2	4.1 (High Risk)
Medium to high yield (5~8)	35%	15%	6.3	2.3 (Medium risk)
Medium (3~5)	22%	40%	3.9	1.2 (Low risk)
Low yield (<3)	15%	40%	1.8	0.6 (lowest risk)

Radical vs Conservative Style: A Comparison of Strategy Indicators

style indicator	Radical style (black)	Conservative style (white)
Average number of active three events per game	6.2	2.2
Average number of times to reach the fourth round per game	3.3	0.9
Average number of active sessions per game	1.5	0.4
Average defensive possessions per game	4.1	9.8
Average number of risky spells per game (variance> threshold)	5.7	1.2
Average return function value (aggressive)	8.3	3.1

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style indicator	Radical style (black)	Conservative style (white)
Average return function value (defensive)	2.9	7.5
Final win rate (first player)	77%	18%
Final win rate (afterhand)	-	23% (if the order of exchange is reversed)

Statistical Table of Nash Equilibrium Achievements

balanced type	frequency of occurences	average round	explain
pure strategy Nash equilibrium	4.6 per game	Distributed globally	If the "three" must be blocked...
mixed strategy equilibrium	1.2 per game	Mid-to-late (20-30 turns)	Black's multi-line attack leaves White unable to defend fully.
locally optimal response cycle	3.5 per game	Midgame (15-25 turns)	The two sides have reached a stable agreement on certain issues.
global convergence to Nash equilibrium	77% of matches	approximately 37.5 loops	The system is in a stable black win state.
Global Convergence to Nash Equilibrium	18% of the matches	approximately 42.3 loops	The white pieces take advantage of the black pieces' mistakes to counterattack.

Key conclusions (numerical analysis)

Winning probability distribution: The aggressive style demonstrates significantly higher success rates (77% vs 18%) when taking the initiative, highlighting the synergistic effect between first-mover advantage and aggressive strategies.

Average wins: 37.5 rounds for aggressive style, 42.3 rounds for conservative style.

Aggressiveness indicator: The number of aggressive style's live three and rush four is significantly higher than that of conservative style, which is consistent with the strategy setting.

Risk exposure: The aggressive style demonstrates a higher proportion of trades in the high-yield range, but also carries significantly greater variance (risk).

The Nash equilibrium is realized: most games converge to the Nash equilibrium state of “black wins”, which is in line with the game prediction under perfect rationality.

Simulation Case 2: Go (Conservative vs. Offensive)

Game theory modeling approach

Radical style: Prioritize high-yield, high-risk moves, such as large-scale operations, intense battles, and actively provoking conflicts.

Conservative style: choose low risk and high certainty moves, such as taking the ground first, avoiding war, and stabilizing the shape.

In the Nash equilibrium framework, each available move in a given scenario corresponds to a specific strategy, with the payoff evaluation function varying according to the player’s style.

U-radical (m) = w1·external potential + w2·attack gain − w3·field loss

U-conservative(m)=w1′·real increase +w2′·safety connectivity −w3′·external potential loss

Summary Table of Numerical Results from Multiple Experiments

Simulation settings:

Board size: 19×19

Number of games: 50 (Aggressive Black vs Conservative White)

Stip: 6.5 (Black Stip)

The game is decided after 200 moves (by point difference).

Fixed style parameters: aggressive weight [0.6,0.3,0.1] conservative weight [0.7,0.2,0.1]

trial batch	Black's win rate	White win rate	Average eye offset (black-to-eye)	average number of hands	Attack count by the radical side	Conservative team's defensive counterattacks
1-10 games	50%	50%	+0.3 mesh	187	12.5	8.2
11th to 20th innings	40%	60%	-2.1 eyes	192	13.1	9.0
21-30 innings	70%	30%	+4.8 mesh	185	14.3	7.5
31-40 innings	60%	40%	+1.5 mesh	190	11.8	8.8
41-50 innings	50%	50%	-0.2 mesh	188	12.6	8.4
Total/Average	54%	46%	+0.86 mesh	188.4	12.86	8.38

radical vs conservative strategy indicator comparison table

style indicator	Radical style (black)	Conservative style (white)
Average number of eyes per field in each session	42.3-mesh	45.8 mesh
Average offside value per game (converted value)	18.5 mesh	12.1 eyes
Average number of attacks per session	12.9 times	5.2 times
Average defensive/counteraction per game	9.8 times	14.6 times
Average number of battles per game	3.2 times	1.8 times
Average per game large field selection revenue valuation	6.7 eyes/hands	4.9 eyes/hands
Average per game risk hand (return variance> threshold)	7.5 hands	2.3 Hand
Final win rate (after handicap)	54%	46%

Statistical Table of Nash Equilibrium Achievements (Local/Global Go)

balanced type	frequency of occurences	Average completion time	explain
local optimal response (formal choice)	15.2 per game	Layout to mid-level	Both sides respond according to the established pattern or the locally optimal response.
global equilibrium (no better strategy for either player)	100% match	Endgame (188 moves)	system convergence to final state
Mixed strategy equilibrium (multiple-choice point with no difference)	2.1 per game	Mid-sized (70-120 lots)	No obvious superiority or inferiority in the trade-off between external force and actual force
The radical side was forced to turn to the conservative side.	1.8 per game	Middle and back positions (130-160 lots)	Failed to attack, switch to closing
The conservative forces were forced to engage in battle.	1.2 per game	Medium (80-110 lots)	When there are not enough resources on-site, proactively seek out opportunities to participate in the task.

Risk-return evaluation table (based on piecewise function model)

Profit range (converted per lot)	proportion of radical style choices	conservative style selection ratio	average income	profit variance (risk)
High yield (>8 eyes)	25%	10%	10.2 mesh	6.5 (High Risk)

Medium to high yield (5~8 eyes)	35%	20%	6.3 eyes	3.2 (Medium to High Risk)
Medium yield (3-5 eyes)	25%	40%	3.8 mesh	1.8 (moderate risk)
Low yield (<3 stars)	15%	30%	1.5 mesh	0.7 (low risk)

Analysis of the Influence of Style on the Winning Rate (6.5 Points)

Style Combination	Black's win rate	White win rate	mean parallax	explain
Radical vs Conservative (Black moves first)	54%	46%	+0.86 mesh	Radical is slightly better but not significantly so
Conservative vs. Radical (Black moves first, then Conservative)	48%	52%	-1.2 mesh	Conservative first moves may result in a slight disadvantage.
Radical vs Radical	58%	42%	+3.5 mesh	More battles mean greater first-move advantage
Conservative vs. Conservative	50%	50%	-0.3 mesh	A steady finish with little impact on the margin

Key conclusion (Go simulation)

The aggressive style holds a slight edge under the point system (54% win rate), though the advantage is not statistically significant, as high-risk play may lead to errors. The average binocular disparity is close to the binocular value (+0.86 binoculars), which indicates that the two styles are close to equilibrium under perfect rationality. The radical side has higher external value but lower practicality, while the conservative side has solid practicality but insufficient external value. The Nash equilibrium is frequently reached locally (as a fixed pattern), and the global game converges to the equilibrium state. The risk-return model reveals that aggressive investors tend to favor high-yield, high-risk strategies, while conservative investors exhibit the opposite preference.

5. Conclusion

This essay proposes a systematic “Nash equilibrium rapid solution”, which aims to provide an efficient and clear solution and risk assessment framework for complete information static games.

This research provides a solution tool for game theory, reduces the dependence of traditional game models on completely rational assumptions, and improves the application ability of theory in complex dynamic situations. At the practical level, the simulation results not only verify the frequent appearance and convergence characteristics of Nash equilibrium in real games, but also deepen the understanding of strategy style, rational boundaries and system stability through quantitative risk-return comparison.There are still something left to improve. First of all, finite rational or behavioral game elements can be introduced to enhance the model’s explanatory power of realistic decision-making. In the subsequent research, the use of machine learning or data-driven methods for parameter adaptive optimization could decrease the subjectivity of the return function and weighted parameters. Moreever, the method can also be expanded to strategic interaction scenarios in a wider range of fields such as economic competition, traffic scheduling, network security.

6. References

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