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The Rossby Number: How Rotation Affects Ocean Circulation

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Abstract:

This project investigates how the Rossby number influences ocean circulation. The Rossby number, defined as $Ro = U/(fL)$, compares inertial forces to Coriolis forces and determines whether rotation dominates a given flow. Through non-dimensionalization of the Navier-Stokes equations, Ro emerges naturally as the ratio of these two forces. The same scale analysis also yields the Ekman number $E = \nu/(fL^2)$, which measures the importance of viscous effects relative to rotation. We systematically define Ro and examine three regimes: $Ro \ll 1$, $Ro \sim 1$, and $Ro \gg 1$. Each regime corresponds to different ocean circulation behaviors, from rotation-dominated large-scale currents to rotation-negligible small-scale motions. A typical mid-latitude ocean current yields $Ro \sim 0.001$, confirming that rotation is the dominant control in large-scale ocean dynamics.

Keywords: Rossby number, Ekman number, Ocean circulation, Coriolis force, Navier-Stokes equations, Scale analysis, Geostrophic balance

1 Introduction

The oceans cover most of the Earth's surface area, roughly 71%. They play an extremely important role to the Earth's climate. Through large-scale circulation, the ocean transports heat from the equator toward the poles, absorbs atmospheric carbon dioxide, and buffers the effects of climate change. Among the many factors that influence ocean circulation, this project focuses on one of the most fundamental: Earth's rotation.

As the planet spins, any moving fluid on its surface experiences an apparent deflection known as the Coriolis force. Whether rotation dominates ocean dynamics depends not only on the Earth's spin but also on the flow's own velocity and length scale. This relationship is captured by the Rossby number Ro. A small Ro indicates that rotation dominates; a large Ro means rotation can be neglected. Therefore, the Rossby number acts as a control parameter that determines which physical processes must be included in a dynamical model. Another important dimensionless number in rotating fluids is the Ekman number E, which measures the importance of viscous effects relative to rotation will also be introduced. For large-scale ocean flows, E is typically very small, so friction can be neglected in the interior.

This project first introduces the Coriolis force, then defines the Rossby number and examines the three cases of the Rossby number to see how the flow behavior changes when Ro is approximately one, much smaller or much larger than one. Finally, we further explore how to derive the Rossby number and Ekman number by non-dimensionalizing the Navier-Stokes equations.

2 Coriolis Force

To understand the Rossby number, we must first understand the Coriolis force. This force is not a real force in the traditional sense — it is a deflection that arises when we describe motion in a rotating reference frame, such as the surface of the Earth.

Imagine standing at the North Pole and throwing a ball straight toward the equator. From your perspective, the ball travels in a straight line. However, an observer in space would see the Earth rotating beneath the ball, causing it to curve relative to the surface. This apparent curvature is the Coriolis effect. In the ocean, the same principle applies: water parcels move in straight lines under their own inertia, but the Earth rotates beneath them, making their paths appear curved.

Mathematically, the Coriolis acceleration (per unit mass) is expressed as:

$$a_{cor} = -2\mathbf{\Omega} \times u$$

where $\mathbf{\Omega}$ is Earth's rotation vector and u is the fluid velocity observed from the rotating frame [1]. This expression comes from the transformation of the momentum equation into a rotating reference frame. The factor 2 appears because both the change in direction of the velocity vector and the rotation of the reference frame contribute to the apparent acceleration. Equivalently, the Coriolis force on a parcel of mass M is $F_{cor} = -2M\mathbf{\Omega} \times u$. The Coriolis force is always perpendicular to the velocity, and therefore does no work on the fluid since the force is perpendicular to the direction of motion.

In large-scale ocean flows, it is often convenient to introduce the Coriolis parameter:

$$f = 2\Omega sin\varphi$$

where φ represents latitude and $\Omega = |\mathbf{\Omega}| = 7.2921 \times 10^{-5}\,\mathrm{rads}^{-1}$.

The Coriolis parameter f varies with latitude — it is zero at the equator and reaches its maximum magnitude at the poles. Since the ocean is very thin compared to its horizontal extent (depth-to-width ratio about 0.001), the vertical component of motion is much smaller than the horizontal components. Under this approximation, only the horizontal component of the Coriolis force is retained:

$$-2\mathbf{\Omega} \times u \approx -f \times u = -fve_x + fue_y$$

where $f = fe_z$ is the Coriolis parameter vector and e_z is the local vertical unit vector, u and v are the eastward and northward velocity components respectively.

However, the Coriolis force does not always matter. For a fast, small-scale flow — such as a local turbulent eddy, the deflection may be too weak to be noticed. For a slow, basin-wide current, the Coriolis force is the dominant control. This is precisely the question that the Rossby number answers: when is the Coriolis force important, and when can we ignore it?

3 Rossby Number

Firstly, we define Rossby number as:

$$Ro \sim \frac{U}{fL}$$

where U and L are characteristic velocity and length scales of the flow, respectively, and f is the Coriolis parameter introduced in the previous section [2]. It represents the ratio of the inertial term to the Coriolis term in the horizontal momentum equation. The inertial acceleration scales as U^2 / L, while the Coriolis acceleration as fU. Thus, when this ratio is small, rotation dominates; when it is large, inertia dominates and rotation can be neglected.

Physically, inertia tends to keep a fluid parcel moving in a straight line, while the Coriolis force tends to deflect it. Whether the deflection is significant depends on how long the flow has to "feel" the rotation. A fast-moving flow with a small length scale will cross a region before rotation can noticeably curve its path; a slow-moving flow with a large length scale will be continually deflected. The Rossby number captures this balance quantitatively.

We will discuss three cases based on the value of Ro. We denote F_{cor} as the Coriolis force and $F_{inertial}$ as the inertial force.

3.1 $Ro \ll 1$: Rotation-Dominated Flow

When $Ro \ll 1$, $F_{cor} \gg F_{inertia}$. The flow is said to be in geostrophic balance, where the pressure gradient force is balanced by the Coriolis force. In this regime, rotation cannot be neglected. Large-scale ocean currents such as the Gulf Stream and the major ocean gyres fall into this regime. Their Rossby numbers are usually of order 10^{-3} to 10^{-1}.

For a typical large-scale ocean current, $U \sim 0.1\mathrm{ms}^{-1}$, $L \sim 10^6\mathrm{m}$, and at mid-latitudes $f \sim 10^{-4}\mathrm{s}^{-1}$. This gives

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$Ro \sim 0.001$, confirming that rotation plays an important role in large-scale ocean circulation [3].

3.2 $Ro \sim 1$: Transitional Flow

When $Ro \sim 1$, $F_{cor} \sim F_{inertial}$. The two forces are comparable, leading to complex, nonlinear dynamics. This case is often observed in mesoscale ocean features such as eddies and fronts, where the flow is strongly ageostrophic and time-dependent.

3.3 $Ro \gg 1$: Inertia-Dominated Flow

When $Ro \gg 1$, $F_{cor} \ll F_{inertial}$. The Coriolis force is too weak to change the flow direction. Rotation becomes negligible, and the flow behaves like a non-rotating fluid dynamics. This case is typical of small-scale or high-speed flows.

Beyond its formal definition, the Rossby number carries a clear physical meaning: it measures how many turnover times of the flow occur within one inertial period. The inertial period $T_f = 2\pi / f$ is the characteristic time scale of rotation. The time scale of the flow is $T_U = L / U$. Their ratio gives $Ro \sim T_f / T_U$ up to a factor of 2π. Thus, a small Ro means that the flow evolves slowly compared with the rotation period, so the Coriolis force has enough time to act and deflect fluid parcels significantly. A large Ro means that the flow changes so rapidly that rotation cannot keep pace.

This interpretation has practical consequences for ocean modeling. When $Ro \ll 1$, the momentum equation can be simplified by dropping the inertial terms, leading to the geostrophic approximation. This reduction is beneficial for computational efficiency: global ocean climate models often exploit this balance to filter out fast gravity waves, allowing longer time steps. Conversely, when $Ro \sim 1$ or larger, such simplifications fail, and the full primitive equations must be solved – which is computationally expensive but necessary for resolving eddy-rich regions like the Gulf Stream separation zone or the Antarctic Circumpolar Current.

The Rossby number also helps guide field measurements. In low-Ro flows, pressure or density gradients can directly reveal the velocity field via geostrophic balance, but in high-Ro flows velocities must be measured directly since inertial effects are no longer negligible. Knowing Ro in advance therefore helps researchers choose suitable sampling strategies and instruments.

Moreover, Ro varies not only with flow type but also with latitude. Since f vanishes at the equator, Ro becomes infinite there regardless of flow speed. At high latitudes, f is large, so even moderate currents yield very small Ro, making polar oceans particularly rotation-controlled.

Finally, the Rossby number is not just a theoretical concept — it has a practical role in classifying ocean flows. By simply estimating U, L, and f, one can quickly determine whether rotation is important for a given phenomenon. This quick assessment helps oceanographers decide which physical processes to include in their analyses and which approximations are reasonable. Thus, the Rossby number serves as a useful starting point for understanding and modelling the diverse motions observed in the ocean.

4 Derivation from the Navier-Stokes Equations

In the previous section, we defined the Rossby number based on a scaling argument. We now show how it emerges directly from the Navier-Stokes equations.

The horizontal momentum equation in a rotating reference frame can be written as:

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u + fk \times u = -\frac{1}{\rho} \nabla p + \nu \nabla^2 u$$

where u is the horizontal velocity vector, f is the Coriolis parameter, ρ is density, p is pressure, and ν is viscosity [3].

The term $(u \cdot \nabla) u$ is the inertial acceleration. It represents the tendency of the flow to maintain its motion due to inertia. Its magnitude scales as U^2 / L, where U is a characteristic velocity and L is a characteristic length scale of the flow. The term $fk \times u$ is the Coriolis acceleration. It represents the deflection caused by Earth's rotation, and its magnitude scales as fU.

Taking the ratio of these two terms gives:

$$\frac{\text{inertialterm}}{\text{Coriolisterm}} \sim \frac{U^2 / L}{fU} = \frac{U}{fL} = Ro$$

This is the Rossby number. It appears naturally when we compare the relative sizes of the inertial and Coriolis terms in the momentum equation. When Ro is small, the Coriolis term dominates and rotation must be included in the model. When Ro is large, the inertial term dominates and rotation can be neglected.

The other terms in the momentum equation can also be scaled. The pressure gradient term $-\frac{1}{\rho} \nabla p$ is what drives the flow, and the viscous term $\nu \nabla^2 u$ is usually small for large-scale ocean flows. In the ocean interior, viscosity is often neglected, and the dominant balance is between the Coriolis force and the pressure gradient — this is the geostrophic balance discussed in Section 3.1.

This derivation shows that the Rossby number comes directly from the momentum equation.

Furthermore, the viscous term $\nu\nabla^2 u$ has magnitude $\nu U / L^2$, and its ratio to the Coriolis term defines another dimensionless number — the Ekman number

$$\frac{\text{viscousterm}}{\text{Coriolisterm}} \sim \frac{\nu U / L^2}{fU} = \frac{\nu}{fL^2} = E$$

For large-scale ocean flows, E is typically very small, so viscous effects can be neglected in the interior [4]. For a mid-latitude ocean, a typical value of kinematic viscosity is $\nu \sim 10^{-6}\,\mathrm{m^2s^{-1}}$. With $f \sim 10^{-4}\,\mathrm{s^{-1}}$ and $L \sim 10^6\,\mathrm{m}$, we get: $E \sim 10^{-14}$. This extremely small value confirms that viscous effects are indeed negligible in the ocean interior, except in thin boundary layers near the surface and sea floor. Thus, for most of the ocean interior, the dominant balance is between the inertial terms and the Coriolis term, and the Rossby number is the key parameter that determines which one wins.

5 Summary

This project introduced the Rossby number and examined its role in ocean circulation. We first discussed the Coriolis force and showed that it arises from describing motion in a rotating reference frame. The Coriolis force deflects moving water to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. It does no work because it is always perpendicular to the velocity.

We then defined the Rossby number as $Ro = U / fL$, where U is a characteristic velocity, L is a characteristic length scale, and f is the Coriolis parameter. The Rossby number compares inertial forces to Coriolis forces. Three cases were discussed. When $Ro \ll 1$, the Coriolis force dominates and the flow is in geostrophic balance. Large-scale ocean currents such as the Gulf Stream fall into this case. When $Ro \sim 1$, both forces are comparable, leading to complex dynamics seen in eddies and fronts. When $Ro \gg 1$, inertial forces dominate and rotation can be ignored, as in small-scale turbulence.

Finally, we showed how the Rossby number emerges from the Navier-Stokes equations through scale analysis. Taking the ratio of the inertial term to the Coriolis term gives $Ro = U / fL$ again. This confirms that the Rossby number is a natural consequence of the momentum equation, not an arbitrary parameter. We also introduced the Ekman number $E = \nu / \left(fL^2\right)$, which compares viscous and Coriolis forces. In the ocean interior, E is extremely small (typically $\sim 10^{-14}$), so viscous effects can be neglected except in thin boundary layers near the surface and sea floor.

The Rossby number is useful because it tells us when rotation matters. For large-scale ocean circulation, Ro is small and rotation plays an important role. For smaller or faster flows, Ro is large and rotation can be neglected. Understanding this helps us decide which processes to include in ocean models and how to interpret ocean observations.

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